EQUITY COMPOSITION AND THE GEOMETRY OF VALUATION

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Abstract

Abnormal-earnings models aggregate retained earnings with contributed capital, treat reallocation as frictionless, and earnings as exogenous. Personal taxation breaks all three. Under the U.S. distribution ordering rule, contributed capital cannot be returned tax-free until all retained earnings have been distributed as taxable dividends. Usually read as burying contributed capital, the rule is what makes the mix matter: reaching that capital requires taxing every retained dollar in front, so equity composition governs divestiture and expected earnings. Two further results are sharp. Optimal payout is discrete: divest below the after-tax hurdle, then stop or clear all retained earnings, so otherwise identical firms diverge. And firm value kinks at the threshold, bending convexly under threshold uncertainty, with a slope change equal to the distribution-tax rate, independent of return parameters. Residual income valuation survives; what fails is the sufficiency of aggregate book value, which conceals a priced state variable at the distribution margin.

Keywords: Residual income valuation; Equity composition; Payout policy; Capital allocation
JEL: M41, G12, G32, G35

I. INTRODUCTION

The balance sheet reports retained earnings and contributed capital separately. Residual income valuation sets that split aside: Ohlson (1995) collapses shareholders' equity into a single book value figure, and the abnormal-earnings literature built on his model has largely aggregated the two ever since. This paper shows the split can be priced. The aggregation is not a harmless simplification, and the information it discards already sits on the face of the statements. What makes the split matter is a feature of the tax law.

That feature is the distribution ordering rule: for ordinary distributions, a U.S. firm cannot return a dollar of contributed capital to its shareholders until every dollar of retained earnings has been distributed and taxed as a dividend.[1] The rule is usually read as burying contributed capital. Since retained earnings stand in front, every distribution is effectively a dividend, and the split between the two looks like mere labeling. This paper reaches the opposite conclusion. Without the rule, contributed capital would come out first, so reaching the tax-free layer would cost nothing, and the split would come far closer to the mere labeling it is usually taken for. Two institutions are therefore at work, and separating them is what distinguishes the mechanism here from the capitalization literature: differential taxation creates the wedge between the two components, while the ordering rule is what forces a firm to pay it to reach the cheaper one.

To see what the rule does, we work with a deliberately ordinary firm. Its book equity divides into retained earnings and contributed capital, and it holds a set of projects it can keep or divest, returning the freed capital to shareholders. The two components of equity leave the firm on different tax terms: retained earnings exit through the dividend tax, contributed capital exits tax-free. The question we ask is what that asymmetry does to the firm's decisions, to the earnings those decisions generate, and finally to price.

The standard abnormal-earnings model, a workhorse of accounting valuation, cannot ask that question, because three of its assumptions rule it out. It treats book equity as homogeneous, so the split between retained earnings and contributed capital cannot matter; it treats reallocation of capital as frictionless, so returning capital costs nothing; and it treats expected earnings as exogenous, so the firm's decisions cannot move them. Personal taxation breaks all three.

[1] We model the ordinary-distribution ordering rule as binding, while recognizing that exchange-treatment repurchases, tax-free separations, and complete liquidations create partial avoidance margins. Throughout, accounting retained earnings serves as the proxy for tax earnings and profits, the account through which the rule formally operates. Appendix A details these provisions.

The first effect of this tax asymmetry between retained earnings and contributed capital is behavioral, and it is smooth. Before the ordering rule enters, small differences in equity composition move the firm's decisions, and its value, by small amounts. Cash inside the firm is fungible, and the tax leaves that untouched; it binds at the exit, where a distributed dollar of retained earnings is a taxable dividend. The cost of freeing a dollar for shareholders therefore depends on the firm's equity composition, and that cost sets the hurdle for keeping the marginal project.

Consider two firms with identical book value and identical projects, one financed by retained earnings, the other by contributed capital; we return to this pair throughout. Each would gain by divesting its weakest assets, but the retained-earnings firm must route every returned dollar through the dividend tax, forfeiting a fraction equal to the dividend tax rate that its contributed-capital twin keeps. When that price is high enough, the retained-earnings firm rationally declines a reallocation of capital that its contributed-capital twin undertakes.

The two firms then hold different project portfolios: the retained-earnings firm keeps its weak projects and earns their low returns, while its twin has shed them and redeployed at the higher outside return. That difference reaches price. The contributed-capital twin is smaller, having returned more capital, but dollar for dollar of book equity it delivers more after-tax value, so the same equity is worth more in its hands. The gap is not an arbitrage: both prices are right, because the two firms generate different after-tax flows, so there is nothing to trade away.

Expected earnings are therefore endogenous to equity composition, and that endogeneity is the entire effect: composition matters here only through the projects the firm keeps. Hold expected earnings fixed and the channel is assumed away, so composition drops out of value. That is the equity composition irrelevance result of Guenther and Sansing (2006), recovered here as exactly that special case.

The rule's second effect is behavioral as well, but sharp rather than smooth. Under the ordering rule the firm's optimal distribution policy is discrete. It divests projects earning below its after-tax hurdle rate, then either stops or clears all retained earnings to reach its contributed capital, never an amount in between. The choice is all-or-nothing because an intermediate distribution pays the tax toll without reaching the tax-free layer: all of the cost, none of the benefit. Otherwise identical firms therefore sort into different payout regimes by equity composition alone. Those with shallow retained earnings may clear them and access contributed capital; those with deep retained earnings may optimally remain in retention.

The third effect is geometric. Value inherits the discreteness of distribution policy, but as a corner, not a cliff: at the threshold, stopping and clearing retained earnings are worth the same, so value stays continuous while its slope changes. Plotted against equity composition, firm value follows two straight lines that meet at the regime threshold (Figure 1). The slope change at that kink equals the distribution-tax rate itself, up to roughly twenty cents of firm value per dollar of retained earnings at current federal statutory rates, which serve as upper-end calibration points for the effective rate.

The firm's return parameters set where the threshold sits, but not how large the slope change is. What investors can see determines the shape the change takes: the geometry so far assumes they observe the threshold exactly, and when they do not, the corner prices as a smooth convex bend with the same total slope change.

Everything above runs through the dividend tax, so share repurchases, taxed as gains rather than as dividends, look like the obvious escape. The escape is shallow. A repurchase returns basis free of tax and taxes the gain, so it changes who bears the tax and when, not how much value is taxed. Summed over the chain of holders, from issuance to the final return of capital, the amounts taxed come to the value that left the firm less the capital paid into it, whatever the mix of dividends and repurchases: shareholders as a group are eventually taxed on everything they did not pay in, and what they paid in is contributed capital. Repurchases therefore attenuate the effective rate through timing without eliminating the wedge; Section V.C makes this precise.

The hurdle-rate differential at the center of this mechanism is not new. Public economics established it decades ago: personal taxation lowers the cost of capital for retained earnings relative to new equity (Auerbach, 1979; Sinn, 1991; Zodrow, 1991). Those models, however, treat the two sources as mutually exclusive alternatives for funding new investment, and they sit outside accounting valuation. This paper carries the established wedge into Ohlson's (1995) framework, where the two are not new-investment alternatives but coexisting components of book equity, and where the ordering rule governs how each exits.

Within accounting valuation, the closest antecedent is Harris and Kemsley (1999), who derive a tax-adjusted abnormal-earnings equation in which equity composition affects normal earnings: retained earnings carry a lower cost of capital than contributed capital. Their expected earnings, however, are exogenous, since composition reprices a given earnings stream without changing what the firm does. Endogenizing that response, and deriving the geometry it produces, is the step taken here.[2]

A related literature debates whether dividend taxes are capitalized into prices at all (Harris, Hubbard, & Kemsley, 2000; Collins & Kemsley, 2000; Dhaliwal, Erickson, Frank, & Banyi, 2003; Hanlon, Myers, & Shevlin, 2003; Guenther & Sansing, 2006). That debate remains

[2] Guenther and Sansing (2006) identify this "indirect" channel, by which equity composition affects value through investment rather than through the direct capitalization channel, and develop it to some extent. It does not, however, carry through to their conclusions, which turn on the direct channel and hold that value is independent of equity composition. The present paper takes up the indirect, capital-allocation path and characterizes the regime structure and valuation geometry it produces under the distribution ordering rule.

unsettled, and nothing here turns on it: the mechanism requires only a positive effective distribution-tax rate.

The payout literature has already documented the regularity this mechanism explains: DeAngelo, DeAngelo, and Stulz (2006) show, in reduced form, that the earned versus contributed mix of equity predicts dividend behavior.[3] What has been missing is the mechanism and the pricing geometry behind that regularity. Composition matters broadly for payout, and the model's distinctive predictions concentrate where distributions reach the return-of-capital margin. The contribution here is to supply the mechanism, not to discover that composition matters.

The analysis is comparative statics: the twin-firm comparison already displayed, with no law of motion required. Dynamics would let firms time their access, but the rule poses the same all-or-nothing choice every period, so deferral moves when the toll is paid, not whether it is paid. Timing is left to companion work.

Broader tax evidence is consistent: the 2003 U.S. dividend-tax cut moved payouts sharply while aggregate investment was unchanged, and dividend taxes reallocate capital across firms according to their access to internal funds (Chetty & Saez, 2005; Becker, Jacob, & Jacob, 2013; Alstadsæter, Jacob, & Michaely, 2017; Yagan, 2015). The first direct tax on the repurchase channel points the same way, since the 2022 excise tax reduced buybacks without an offsetting rise in dividends and affected firms accumulated cash rather than changing investment (Bargeron et al., 2024; Autore, Barnes, Clarke, & Schrowang, 2025). That evidence turns on access to internal funds and on the choice of payout channel. Neither turns on the earned-versus-

[3] The earned/contributed mix of book equity is a first-order determinant of payout: DeAngelo, DeAngelo, and Stulz (2006) find that the propensity to pay dividends rises strongly with retained earnings as a share of total equity, with greater explanatory power than profitability or growth, consistent with a life-cycle in which firms return capital as investment opportunities decline (Grullon, Michaely, & Swaminathan, 2002). These are the firms for which the ordering-rule toll on retained earnings is economically live.

contributed composition of equity, and neither shows the response concentrating at a threshold, the joint signature this model predicts.

Sections II and III set out the firm and derive the smooth benchmark in which equity composition moves expected earnings; Section IV adds the ordering rule and derives the discrete policy; Section V bounds the scope and answers three objections, concerning dynamics, the funding of distributions, and repurchases; Section VI converts policy into the kinked value geometry; Section VII states the observable implications; and Section VIII concludes.

II. THE VALUATION EQUATION UNDER PERSONAL TAXATION

A firm holds equity that is split between retained earnings and contributed capital, and for each dollar of that equity it chooses whether to keep the dollar invested in a project or return it to shareholders. The two components are taxed differently on the way out: a dollar returned from retained earnings is taxed as a dividend, while a dollar returned as contributed capital is not. The cost of returning a dollar therefore depends on where in the firm's equity it sits. This section works out what that asymmetry implies for which projects the firm keeps, and in turn for its expected earnings and its value. Throughout, no sequencing constraint applies, and the firm does not choose which component funds a distribution. Each dollar of book equity is tied to its component and bears that component's tax on the way out. The ordering rule, which forces retained earnings out first, is imposed only in Section IV, so what follows is the smooth benchmark that precedes it.

A. Basic Assumptions

We begin by adopting the foundational assumptions of Ohlson (1995), including risk neutrality, homogeneous investor beliefs, a flat and non-stochastic term structure of interest rates, and the dividend-discount model of firm valuation. Under these assumptions, valuation can be represented using a single discount rate, which we interpret through a representative investor, an aggregation across investors rather than any single marginal investor.[4] We also incorporate the classical assumptions of Miller and Modigliani (1961), which Ohlson likewise employs. The sharp policy and valuation results below are properties of this stylized benchmark and would soften under risk aversion, a stochastic term structure, or heterogeneous beliefs.

Subsequent research has relaxed many of these assumptions in various directions. We modify a single one, the homogeneity of equity. The consequence is concrete: once retained earnings and contributed capital are taxed differently on the way out, the firm's investment, and with it its expected earnings and its value, comes to depend on which of the two it holds.

Throughout the model, retained earnings denotes the accounting balance, an imperfect proxy for tax-law earnings and profits; Appendix A details where the two diverge. To incorporate investor-level taxation, we modify Ohlson's model such that firm value equals the present value of all expected future after-tax equity distributions:

$$P_t = \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t([(1-\gamma)d_{t+\tau} + cd_{t+\tau}] \tag{1}$$

where:

P_t	= the price of the firm's equity at date t.
d_t	= equity distributions paid from retained or future earnings at date t.
cd_t	= net capital distributions paid from contributed capital at date t.
R_f	= the constant risk-free rate plus one.[5]
$E_t\,[.]$	= the expected value operator conditioned on date *t* information.
γ	= the fixed, uniform, investor-level tax on equity distributions.

[4] The reading follows Guenther and Sansing (2006, 2010), which argue that tax capitalization does not depend on the rate faced by a hypothetical marginal investor, and the portfolio-theory tradition of Brennan (1970) and Gordon and Bradford (1980), in which the taxes of all investors bear on price at once. The effective rate this implies is developed below.

[5] R_f is measured after personal taxes on investment income and is therefore independent of the equity distribution tax γ, which applies only to distributions of retained earnings. Using an after-tax discount rate is standard in valuing after-tax equity distributions (e.g., Auerbach, 1984; Poterba & Summers, 1984; Sinn, 1991; Guenther & Sansing, 2006).

This specification reflects that distributions from retained earnings are taxable, whereas returns of contributed capital are not. Note that although $d_t \geq 0$, cd_t may be positive or negative, capturing both capital contributions and distributions.

We assume a uniform rate γ, with $0 \leqslant \gamma < 1$, though actual rates vary across investors, distribution types, and time, and institutional features can make tax consequences depend on the ordering of distributions. Both are excluded from this benchmark and introduced later. The rate γ is best interpreted as an effective rate on equity distributions rather than a statutory one: a blend across distribution forms and across investor types.[6] Since 2003, qualified dividends and long-term capital gains have generally faced essentially the same preferential statutory rates, and long-term gains are the taxable component of a repurchase; Appendix A.C sets out the governing provisions.

We do not attribute the rate to a single marginal investor. Following the joint-determination view (Brennan, 1970; Gordon & Bradford, 1980; Guenther & Sansing, 2010), all investors bear on price at once, so γ is the clientele-weighted effective rate: tax-exempt investors enter the blend at $\gamma = 0$ and pull the effective rate below the statutory rate, without driving it to zero so long as taxed investors hold weight. All results below require only $0 < \gamma < 1$; Appendix A.E develops the institutional detail behind this effective rate.

We next specify the clean-surplus relations:

$$RE_t = RE_{t-1} + NI_t - d_t;\ RE_{t-1} = RE_t - NI_t + d_t \tag{2a}$$

$$C_t = C_{t-1} - cd_t;\ C_{t-1} = C_t + cd_t \tag{2b}$$

[6] An effective rate can also depart from the statutory rate for a reason outside this benchmark. Kemsley, Sivadasan, and Subramaniam (2018) show that the marginal dividend tax rate is a composite, the nominal rate less the tax gain from leverage a distribution creates, and that this composite governs when a dividend is financed with debt or with internally generated equity. This model holds the corporate layer fixed, so that offset is absent by construction. Were it material, it would lower γ and scale the magnitudes below in proportion, leaving the geometry unchanged.

where:

RE_t = retained earnings at time t.
C_t = contributed capital at time t.
NI_t = net income at time t.

These relations follow Ohlson (1995) but explicitly distinguish between retained earnings and contributed capital. Although dirty-surplus accounting exists in practice (e.*g*., Stark, 1997; Lo & Lys, 2000; Ohlson & Juettner-Nauroth, 2005; Penman, 2005; Lai, 2015), we retain the clean-surplus assumption because it makes book equity divide exhaustively into retained earnings and contributed capital. Items that bypass net income often affect earnings and profits only upon realization, at which point they may pass through income into retained earnings, so the omission primarily affects the timing and measurement of the split rather than the logic of the model.[7]

Multiplying both sides of (2a) by $(1 - \gamma)$ yields the after-tax clean-surplus relation:

$$(1-\gamma)RE_{t-1} = (1-\gamma)RE_t - (1-\gamma)NI_t + (1-\gamma)d_t \tag{2c}$$

B. Abnormal Earnings

We define after-tax abnormal earnings (NI_t^a) as:

$$NI_t^a \equiv (1-\gamma)NI_t - \left(R_f - 1\right)[C_{t-1} + (1-\gamma)RE_{t-1}] \tag{3}$$

This definition expresses earnings on an after-tax basis, so abnormal earnings are measured consistently with investor-level valuation. It adjusts both net income and the normal return on equity for personal taxation, but asymmetrically across equity components: the after-tax opportunity cost of retained earnings is $(R_f - 1)(1 - \gamma)$, while the opportunity cost of contributed capital remains $(R_f - 1)$. Personal taxation therefore creates two distinct effective hurdle rates, a lower one for internally generated equity and a higher one for externally

[7] For a discussion of treasury stock, see Appendix A.

contributed equity. Abnormal earnings are then defined relative to a composite normal return that varies with equity composition, not a single uniform cost of capital.[8]

The remainder of the paper works with a concrete version of this firm, carried through unchanged: book equity BV split into RE and C, a fixed set of internal projects ranked from the weakest to the strongest, and a keep-or-distribute choice for each dollar, with distributed funds reinvested externally at the risk-free rate. Section IV.A states the schedule in parametric form; this section depends only on the spread it creates, that some projects earn more than the risk-free rate and some less.

The choice for each dollar is a comparison of returns. A dollar kept in its project earns that project's return. A dollar distributed from retained earnings is taxed at γ, so the investor reinvests only $(1-\gamma)$ of it and earns $(R_f-1)(1-\gamma)$ per original dollar. A dollar returned from contributed capital reinvests whole and earns R_f-1. The two hurdle rates of equation (3) are these external returns: a retained-earnings dollar is worth keeping only if its project beats $(R_f-1)(1-\gamma)$, and a contributed-capital dollar only if its project beats R_f-1. Absent a sequencing constraint these decisions are separable: each dollar's outside option depends only on which component it belongs to, so the firm may decide dollar by dollar, and the per-dollar comparisons are the whole of the optimal policy.

This comparison ties the firm's investment to its equity composition. Because the hurdle on retained earnings is lower, a firm financed mostly by retained earnings keeps projects that an otherwise identical firm financed with contributed capital would divest. That firms divest at all is not assumed: as Proposition 1 (Baseline Distribution) shows in Section IV, every optimal policy

[8] That internally generated equity is cheaper than externally raised equity when distributions are taxed is a standard result (Guenther & Sansing, 2006; Auerbach, 1979), and the distribution ordering rule extends the same logic to the return-of-capital margin within existing equity. The capitalization is exact for the marginal, normal-return dollar that fixes the opportunity cost, and presumes a going concern that eventually distributes to taxable shareholders.

divests each project earning less than the after-tax hurdle $(R_f - 1)(1 - \gamma)$. How much the firm divests, and so which projects remain and the earnings they generate, therefore varies with equity composition, and here, before the ordering rule enters, it varies smoothly. Section VI.F returns to this benchmark once the rule is in place.

The foundations of this hurdle-rate differential are well established: Auerbach (1979), Sinn (1991), and Zodrow (1991) show that personal taxes reduce the effective cost of retained earnings relative to new equity, an effect Guenther and Sansing (2006) call the "internal corporate investment channel" and that Auerbach (1984) and Becker, Jacob, and Jacob (2013) examine empirically.

Holding expected earnings fixed shuts off this dependence, and equity composition then has no effect on firm value, because the tax discount on the retained-earnings principal and the lower normal-return charge that principal must cover cancel exactly, as Section III.C shows in closed form.[9] This is the composition-irrelevance result of Guenther and Sansing (2006), recovered here as the special case that fixing expected earnings creates. Their conclusion holds precisely when the earnings stream is assumed exogenous, and it is that assumption, not the result, that the remainder of the paper removes.

The $(1 - \gamma)$ capitalization of retained earnings rests on an assumption that deserves a name and a flag.

Assumption (Eventual Taxation): Every dollar of retained earnings eventually bears the distribution tax γ, whether distributed now or retained and distributed later; the firm cannot escape the tax by retaining.

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[9] Replacing one dollar of contributed capital with one dollar of retained earnings, holding expected earnings fixed, lowers the tax-adjusted book-value term by γ. The same substitution lowers the normal-return charge on that dollar from $(R_f - 1)$ to $(R_f - 1)(1 - \gamma)$, raising after-tax abnormal earnings by $\gamma(R_f - 1)$ each period; discounted as a perpetuity at $R_f - 1$, this contributes $\gamma(R_f - 1)/(R_f - 1) = \gamma$ to value. The two effects sum to zero, so equity composition does not affect value when expected earnings are held fixed, consistent with Guenther and Sansing (2006).

The assumption follows Auerbach (1979) and Sinn (1991), and its content is simple. Earnings retained inside the firm face the same distribution tax when they eventually leave, so a retained dollar and a contributed dollar are compared after tax on both sides. We flag the assumption rather than adopt it as settled, and we treat its conditional status as part of the paper's subject. Whether anything in the tax law actually forces the tax to be borne is left open at this point in the analysis, and Section IV returns to it. Nor do repurchases provide an escape from this eventual taxation. A repurchase returns basis free of tax but taxes the gain, and Section V.C shows that the amounts taxed over a complete history sum to the value distributed less the capital paid in, whatever the mix of dividends and repurchases. The taxation half of the assumption is therefore earned rather than assumed: what the assumption continues to carry is that value eventually leaves the firm, and repurchases bear on the effective value of γ rather than on the wedge itself.

C. Valuation of Normal and Abnormal Earnings

We now derive a valuation expression separating after-tax normal and abnormal earnings. Begin by rearranging (2b) and (2c):

$$cd_t = C_{t-1} - C_t \tag{4a}$$

$$(1-\gamma)d_t = (1-\gamma)RE_{t-1} - (1-\gamma)RE_t + (1-\gamma)NI_t \tag{4b}$$

Rearrange the definition of abnormal earnings in (3) to specify net income:

$$(1-\gamma)NI_t \equiv NI_t^a + \left(R_f - 1\right)[C_{t-1} + (1-\gamma)RE_{t-1}] \tag{5}$$

Substitute (5) into (4b):

$$(1-\gamma)d_t = (1-\gamma)RE_{t-1} - (1-\gamma)RE_t + NI_t^a + \left(R_f - 1\right)[C_{t-1} + (1-\gamma)RE_{t-1}]$$

Merge this equation with (4a) to specify total after-tax capital and ordinary dividends:

$$cd_t + (1-\gamma)d_t = C_{t-1} - C_t + (1-\gamma)RE_{t-1} - (1-\gamma)RE_t + NI_t^a + \left(R_f - 1\right)[C_{t-1} + (1-\gamma)RE_{t-1}]$$

Substitute this expression into (1) and simplify:

$$P_t = \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t \left[-C_{t+\tau} - (1-\gamma) RE_{t+\tau} + NI_{t+\tau}^a + R_f [C_{t-1+\tau} + (1-\gamma) RE_{t-1+\tau}] \right]$$

$$P_t = \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t \left[-C_{t+\tau} - (1-\gamma) RE_{t+\tau} + R_f [C_{t-1+\tau} + (1-\gamma) RE_{t-1+\tau}] \right] + \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t [NI_{t+\tau}^a]$$

The first series is telescoping, simplifying to $C_t + (1- \gamma) RE_t$[10], producing:

$$P_t = C_t + (1- \gamma) RE_t + \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t [NI_{t+\tau}^a] \tag{6}$$

Equation (6) is the tax-adjusted analog of the core Ohlson (1995) result, with firm value equal to current after-tax equity plus the present value of expected after-tax abnormal earnings. The structure is preserved, but the content differs. The equity base is no longer homogeneous, and the normal return embedded in abnormal earnings varies with equity composition.

Under the benchmark assumptions, most importantly homogeneous beliefs and a uniform tax rate, the valuation relation remains globally linear in book value and in abnormal earnings, as in Ohlson (1995). That linearity, however, depends on the benchmark assumptions, and it breaks once the ordering rule makes distribution policy discrete in Section IV. Indeed, without a sequencing constraint the tax-adjusted valuation would remain globally linear and the analysis could end here. The ordering rule is what turns the smooth composition effect into the regime structure studied below. A firm free to designate the source of each distributed dollar is a different counterfactual again, and Section VI.F shows that composition then loses its grip on the divestiture boundary altogether.

[10] This simplification requires that $\lim_{\tau \to \infty} R_f{}^{-\tau} E_t[-C_\tau - (1-\gamma) RE_\tau] = 0$. This condition assumes that the firm's net book value does not grow at or above the risk-free rate indefinitely, so that sufficiently distant terms become negligible after discounting. A similar assumption is made in Ohlson (1995), and the introduction of taxes does not alter the underlying intuition.

Finally, substitute equation (3) into (6) to fully express price in terms of equity and abnormal earnings:[11]

$$P_t = C_t + (1-\ \gamma)RE_t + \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t\left[(1-\gamma)NI_{t+\tau} - \left(R_f - 1\right)[C_{t-1+\tau} + (1-\gamma)RE_{t-1+\tau}]\right] \quad (7)$$

Equation (7) states price in terms of current equity and the full path of expected future earnings, with no dynamics yet imposed on that path. Section III closes it by adding an information process, which replaces the path with current observables.

III. THE LINEAR BENCHMARK

A. The Linear Transformation

We now impose Ohlson's (1995) autoregressive structure (see also Feltham & Ohlson, 1995, 1999) on abnormal earnings:

$$NI_{t+1}^{a} = \ \omega NI_t^{a} +\ \nu_t +\ \varepsilon_{1t+1} \quad (8a)$$

$$\nu_{t+1} =\ \lambda\nu_t + \varepsilon_{2t+1} \quad (8b)$$

This stylized process captures the decay of abnormal earnings under competition (Callen & Morel, 2001): ω is the persistence of abnormal earnings, ν represents value-relevant information not contained in them, and the error terms are mean-zero innovations. The persistence parameters ω and λ are exogenous, fixed, and known, with values in [0, 1]; the upper endpoint is admissible and Section VI.C uses it for one component. The process applies to after-tax abnormal earnings as defined in (3), so the personal tax adjustment sits inside the primitive and does not disturb the autoregressive structure.

[11] We derive equation (7) without the simplifying steps used in Harris and Kemsley (1999), but the resulting expression reduces to the same open-form expression as their equation (3), independent of the empirical interpretation they place on it. Harris and Kemsley do not develop a linear representation of the model, which limits the transparency of its valuation implications.

Persistence governs how abnormal earnings are priced, not how the firm behaves. The optimal policy derived in Section IV turns on the tax rate and the return schedule alone, never on ω. The process in (8a) is written for a single autoregressive flow, which suffices while policy is exogenous. Once distribution policy makes part of the flow endogenous to equity composition, Section VI.C separates that part from the competitive remainder and assigns each its own persistence, an extension of (8a) rather than a departure from it.

Imposing the autoregressive structure in (8) on the unrestricted expression in (7) and solving yields a linear pricing relation (see Ohlson, 1995, Appendix 1, for the standard derivation; the inclusion of a constant tax parameter γ does not alter the structure of the solution):

$$P_t = C_t + (1 - \gamma)RE_t + \alpha_1 NI_t^a + \alpha_2 \nu_t \tag{9}$$

where

$$\alpha_1 = \omega/(R_f - \omega) \geq 0$$
$$\alpha_2 = R_f/(R_f - \omega)(R_f - \lambda) > 0$$

Use the definition of abnormal earnings in (3) to substitute for NI_t^a in (9):

$$P_t = C_t + (1 - \gamma)RE_t + \alpha_1(1-\gamma)NI_t - \alpha_1(R_f - 1)[C_{t-1} + (1-\gamma)RE_{t-1}] + \alpha_2\nu_t \tag{10}$$

Use (2a) and (2b) to replace RE_{t-1} and C_{t-1} and simplify:

$$P_t = (1-k)[BV_t - \gamma RE_t] + k[\varphi(1-\gamma)NI_t - (1-\gamma)d_t - cd_t] + \alpha_2\nu_t \tag{11}$$

where BV is the book value of total shareholders' equity, equal to $C + RE$, and we use the following definitions from Ohlson (1995):

$$k = \omega\,(R_f - 1)/\,(R_f - \omega) = \alpha_1(R_f - 1)$$
$$\varphi = R_f/\,(R_f - 1)$$

Equation (11) is the tax-adjusted, linearized representation of equation (7). The parameter φ serves as the earnings multiplier, while k increases with ω, the persistence of abnormal earnings. The term γRE_t inside the book value expression is the paper's central object, making its first closed-form appearance: the tax discount applied to retained earnings, embedding the differential hurdle rates of Section II directly into price. Retained earnings and contributed capital enter with different effective costs of capital, so BV_t alone is no longer a sufficient statistic for expected normal returns.

The structure of the model survives the tax adjustment: a single pricing kernel and, absent institutional ordering features, global linearity in book value and earnings. That linearity is what the rest of the paper works within, and eventually what it breaks.

B. Investor Heterogeneity and the Uniform Tax Rate

The benchmark assumes a representative investor facing the uniform rate γ. This is a deliberate scope condition. Actual investors face heterogeneous rates, and a substantial literature documents that those differences affect equilibrium prices through clientele formation (Elton & Gruber, 1970; Miller, 1977; Graham, 1999; Graham & Kumar, 2006; Desai & Jin, 2011). The uniform rate isolates the allocation mechanism from differential pricing across investor types. Appendix A.E develops the effective rate as a realization-weighted blend across the dividend and repurchase channels, falling as investors' horizons lengthen.

Heterogeneity lies outside the formal model, but the uniform rate has a natural reduced-form reading: when investors face heterogeneous tax rates, the effective rate embedded in valuation is the clientele-weighted blend, and shifts in investor clientele, driven by changes in payout policy or equity composition, move that blend. In the setting of Section IV, different clienteles would face different effective costs of accessing contributed capital under the ordering rule; characterizing that interaction formally is left to future work.[12]

[12] Heterogeneous rates imply investor-specific regime thresholds rather than the single threshold characterized here, and as equity composition shifts the price-setting clientele may change, altering the effective rate embedded in prices and creating multiple potential regime boundaries. Whether distinct regimes attract distinct clienteles is an open question. Such sorting could influence the level of prices through discount rates, but it does not overturn the within-firm capital-allocation mechanism emphasized here. A complete characterization would require an equilibrium model of clientele formation, which lies beyond the scope of this paper.

C. Composition, Normal Earnings, and the Retained Earnings Wedge

The linear representation makes the benchmark's central property easy to state. Equity composition affects value through expected earnings, and through nothing else. The appearance of γRE_t in (11) is not a second channel, because the tax discount on the retained-earnings principal is exactly offset by the lower normal-return charge that principal must cover. Hold expected earnings fixed and composition washes out, the irrelevance recovered as a special case in Section II.

But expected earnings are not fixed, and the chain from composition to value has four links: composition fixes how much of the firm's equity faces the low hurdle and how much the high one; the hurdles determine what the firm divests; the projects that remain determine expected earnings; and expected earnings determine value. So firms identical in every respect except composition hold different portfolios, earn different expected earnings, and carry different values.

None of the four links is a new assumption. The first is definitional, since equation (3) charges retained earnings an after-tax opportunity cost of $(Rf - 1)(1 - \gamma)$ and contributed capital the full $(Rf - 1)$. The second is a claim about behavior, settled in the benchmark by the per-dollar comparison of Section II.B, and Proposition 1 shows the same after-tax floor survives once the ordering rule couples the dollars. The third is the firm's specification, with clean surplus carrying the earnings of surviving projects into retained earnings. The fourth is the valuation relation itself.

The effect is continuous. A small change in composition moves only a small set of dollars from one hurdle to the other, and because the projects form an ordered continuum the divestiture boundary shifts rather than jumps; on the pricing side retained earnings carry the same coefficient $(1 - \gamma)$ in (11) at every level, so nothing in the benchmark creates a threshold. What the benchmark does not deliver is any magnitude, since the schedule is not yet parametric and the optimum has not been derived. Section IV supplies both, and Section VI carries the result into value.

The two hurdle rates differ by $\gamma(R_f - 1)$, the retained-earnings wedge, which measures how much less a retained-earnings dollar must earn to justify staying inside the firm. Because the two components carry different costs, the normal return on the firm's total equity is their book-value-weighted average:

$$(R_f - 1)(1 - \gamma \frac{RE}{BV}) \tag{12}$$

Equation (12) is the composite normal return, and the only firm characteristic that moves it is the retained-earnings share RE/BV. It falls linearly in that share, approaching $R_f - 1$ as the share goes to zero and $(R_f - 1)(1 - \gamma)$ as it goes to one. The wedge sets the range; composition sets the position within it.

The construction will look familiar. It is a weighted average cost of capital, formed across two components of equity rather than across debt and equity, and the analogy brings the standard objection with it. An average governs investment only when the marginal dollar is drawn in the same proportions as the existing stock, and otherwise the marginal source is what matters. The objection does not bite here, because nothing in the model tests a project against the average. The decision margin is the pair of component rates in (3), where a retained-earnings dollar must beat $(R_f - 1)(1 - \gamma)$ and a contributed-capital dollar must beat $R_f - 1$, each dollar against its own source. The average in (12) is what those decisions aggregate to for pricing, the charge the firm's book must cover before any abnormal earnings accrue. The retained-earnings share drives the decisions and the charge alike. A firm holding more retained earnings has more dollars tested against the lower hurdle and fewer against the higher one, so it keeps projects a contributed-capital firm would divest, while the same share lowers the composite charge. Expected earnings therefore depend on equity composition even in the benchmark, a contrast with prior frameworks, including Ohlson (1995), that treat earnings as exogenous primitives.[13]

A corollary concerns payout. Within the benchmark a dollar of retained earnings distributed as a dividend reduces firm value by $(1 - \gamma)$ while a dollar of contributed capital returned reduces it by one, so the distribution tax makes payout non-neutral with respect to equity composition. Offsetting equity issuance cannot undo the difference: a new dollar enters as contributed capital, and no issuance rebuilds retained earnings, which only earning and retaining can create. A dividend paired with an offsetting issue therefore restores the firm's size but not its composition; it converts a retained-earnings dollar into a contributed-capital dollar, with the distribution tax paid on the way.[14]

That is the benchmark in full: equity composition matters, but only through expected earnings, and only smoothly. The linearity is global. Whether it survives once the firm must cross a taxed layer to reach its contributed capital is what the ordering rule settles.

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[13] Hanlon, Myers, and Shevlin (2003) show that if expected future earnings are fixed and known, price is orthogonal to the mix of retained earnings and contributed capital. Here the normal-earnings benchmark in equation (11) varies with that mix, and Section VI.F establishes the fixed-earnings results as nested cases.

[14] In standard tax-capitalization models, dividend displacement is associated with payout neutrality, since a distribution can be offset by new equity issuance that leaves investor after-tax wealth unchanged. Displacement survives here in that form. A distributed retained-earnings dollar reduces firm value by $(1 - \gamma)$, exactly what the investor receives after tax, so the payout event itself is priced neutrally. The composition change left behind has a direction. The round trip lowers RE/BV, raises the composite normal return of equation (12), and shifts dollars from the lower hurdle to the higher one. The firm then divests more, and the dollars it sheds can leave untaxed at $R_f - 1$ rather than sitting in projects that earn less. The gain is paid for by the γ collected on the dividend leg, so the round trip poses the same comparison that the access decision of Section IV resolves. That toll is also what gives the ordering rule its force, since costless conversion of retained earnings into contributed capital would reproduce the free-designation case of Section VI.F, in which composition loses its grip on the divestiture boundary.

IV. CAPITAL ALLOCATION UNDER THE DISTRIBUTION ORDERING RULE

Section II introduced a firm that holds book value BV, split between retained earnings RE and contributed capital C, invests it across a schedule of projects ordered by their rates of return, and chooses, dollar by dollar and independently, whether to keep each unit invested or return it to shareholders. This section places that firm under the distribution ordering rule and derives its optimal distribution policy.

A. The Formal Setup

The setting requires only elements already introduced. The firm is controlled by the representative investor of Section II.A, who is risk neutral, maximizes total after-tax returns, and chooses a single distribution level d between zero and BV. Distributed funds are reinvested externally at the risk-free rate, so each dollar reaching the investor earns Rf − 1, and the distribution tax determines how much of a distributed dollar reaches him: distributions drawn from retained earnings are taxed at γ, those drawn from contributed capital are not. The ordering rule binds, so contributed capital can be reached only after all retained earnings have been distributed. And the firm divests its least productive projects first, since shedding a weaker project while keeping a stronger one dominates the reverse, so a distribution of size d removes the d lowest-return projects.

The rule has two immediate consequences. It removes the separability of Section II.B, since the cost of reaching a contributed-capital dollar now depends on how much retained earnings remains inside the firm, so the dollars are coupled and the firm can no longer decide one at a time. And a retained-earnings dollar cannot leave untaxed: sequencing defers the tax but does not avoid it. That discharges the Eventual Taxation assumption of Section II, which under the rule is no longer an assumption but a property of the exit itself. Repurchases, the apparent exception, are taken up in Section V.C, where the same conclusion is reached without the rule, since the amounts taxed over a complete history sum to the value distributed less the capital paid in whichever instrument carries it.

The cost of that sequencing depends on composition. To reach its contributed capital, a firm holding eighty percent of book equity as retained earnings must first push all eighty through the tax, while a firm holding twenty percent pays on only twenty. The rule is the same for both; the toll is not. The illustration at the end of this section returns to these two firms.

Formally, the firm has book value $BV > 0$, divided into retained earnings RE and contributed capital $C = BV - RE$, with $0 < RE < BV$. The investor chooses a distribution $d \in [0, BV]$; index each dollar by its position $x \in [0, BV]$ in the ranked schedule of marginal returns, so that x also measures cumulative capital distributed. The ordering rule fixes the after-tax return on each distributed dollar:

After-tax Return on Distributed Capital

$$g(x) = \begin{cases} (R_f - 1)(1 - \gamma) \; if \; x \leq RE \\ (R_f - 1) \; if \; x > RE \end{cases}$$

For $x \leq RE$ the distributed dollar is drawn from retained earnings, is taxed at the rate γ, with $0 < \gamma < 1$, and earns $(R_f - 1)(1 - \gamma)$ externally; beyond RE it is drawn from contributed capital and earns the untaxed $R_f - 1$. The marginal return on distributed funds is therefore nondecreasing in x.

A dollar kept inside the firm earns the return of its project. With full internal investment, the marginal return on retained capital, ranked from the weakest project to the strongest, is

After-tax Return on Retained Capital

$$f(x) = (R_f - 1)(1 - \gamma) - \sigma + \left(\gamma(R_f - 1) + \delta + \sigma\right)\left(\frac{x}{BV}\right)^a$$

with $a > 0$ and $\delta, \sigma \geqslant 0$. Three parameters shape the schedule. The premium δ is how much the best internal project beats the risk-free rate, so the strongest project earns $f(BV) = R_f - 1 + \delta$. The gap σ is how far the weakest project falls below the after-tax hurdle, so the weakest earns $f(0) = (R_f - 1)(1 - \gamma) - \sigma$. The exponent a sets the curvature of the schedule. At $a = 1$ internal returns spread uniformly, at $a > 1$ projects cluster at low returns, and at $a < 1$ they cluster at high ones. The power form is the paper's one parametric object, and it does less work than its prominence suggests: the Remark following Proposition 2 shows the structure of the solution survives on any continuous, strictly increasing schedule, and only Proposition 3's closed forms need the specific shape.

The basis of f deserves explicit statement, because the tax wedge depends on it. f is the after-tax return per dollar of book capital retained. A kept dollar earns $f(x)$ with no further tax on the way out, while a distributed retained-earnings dollar bears the exit tax on its principal, so only $(1 - \gamma)$ of it reaches the external market and it earns $(R_f - 1)(1 - \gamma)$ per original dollar. The tax binds on the principal at the exit, not on the internal flow.

Whether the two hurdles differ at all turns on this convention. If f were a pre-tax flow that also bore γ on the way out, the $(1 - \gamma)$ would cancel from both sides, the hurdles would coincide, and the tax would stop mattering at the retention margin, which is the standard new-view neutrality result. It does not apply here, because the two concern different margins. Neutrality is about the timing of distributions, whether a dollar leaves now or later, and at that margin the tax necessarily cancels. This paper's mechanism is about the return of capital, whether contributed capital can leave tax-free at all, which turns on whether retained earnings have been exhausted first. Cancellation requires the same tax on both sides; here it falls on one side only.[15]

[15] Treating the internal return as untaxed is an extreme version of something real. A dollar left inside the firm bears a lower effective tax rate than a dollar paid out today, because the tax is deferred and because

Given the divestiture order fixed in the setup, total after-tax returns to the investor are

$$TR(d) = \int_{d}^{BV} f(x)dx + \int_{0}^{d} g(x)dx$$

where the first term collects the returns on projects kept inside the firm and the second the external returns on distributed funds. Write $G(d, RE) = \int_0^d g(x)dx$ for the second term. It depends on RE because g steps up at RE, where the draw passes from retained earnings to contributed capital. Since the investor values a dollar of return the same wherever it is earned, the two terms simply add, and the investor chooses d to maximize $TR(d)$. D^* denotes the set of optimal distributions and d^* an element of it.

Two distributions organize the solution. The Baseline Distribution d_{RE} divests every project earning less than the after-tax hurdle, $f(d_{RE}) = (R_f - 1)(1 - \gamma)$. The Access Distribution d_C divests every project earning less than the tax-free external return, $f(d_C) = R_f - 1$. When d_C exceeds RE, undertaking it clears all retained earnings and so gains access to the contributed-capital layer, which is what the name records.

The Baseline Distribution must come entirely out of retained earnings, $d_{RE} \leqslant RE$, since otherwise paying it would itself reach contributed capital and no separate access decision would remain. That holds when σ is small, because σ measures how far the weakest project falls below the after-tax hurdle and so how much the baseline sheds. Propositions 2 through 5 set $\sigma = 0$, emptying the baseline, and Proposition 6 inherits the restriction through the optimized flow it prices; nothing is lost: the baseline solves $f(d_{RE}) = (R_f - 1)(1 - \gamma)$, which does not involve RE,

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capital can be returned through repurchases rather than dividends, an effective rate Appendix A.E develops. The model charges the internal return nothing, which takes the gap to its limit. A smaller gap would produce the same two hurdles separated by a smaller wedge, so the assumption sharpens the mechanism rather than creating it. More generally, if retained internal returns themselves bore an effective shareholder-level burden, the relevant wedge would be the difference between the two effective burdens. The binary structure survives, since it requires only a positive wedge and the layered exit, while the magnitudes would track that smaller wedge rather than the full statutory rate.

so every firm sheds the same amount whatever its composition and that amount nets out of the comparison.

One object carries enough economic content to state in words. The threshold ratio $\theta = \gamma(Rf - 1)/[\gamma(Rf - 1) + \delta]$ is the tax toll on a retained-earnings dollar as a share of that toll plus the premium δ: it rises with the toll, falls as internal opportunities strengthen, and drives both the size of the Access Distribution and the location of the regime threshold. The table collects it with the rest of the notation, including TR*(RE) and AR*, which appear in the results below. Appendix B works in its own notation and records the correspondence.

Notation Summary

Symbol	Meaning
f(x); g(x)	Marginal after-tax return on the dollar at position *x*, retained (*f*) or distributed (*g*)
TR(d); G(d, RE)	Total after-tax returns at distribution *d*; the external-return component of *TR*
d_{RE}; d_C	The Baseline Distribution; the Access Distribution
D; d**	The set of optimal distributions; an element of it
TR*(RE); AR*	Optimized total returns given composition; average internal return on funds remaining after the optimal distribution
*θ; RE**	The threshold ratio; the regime threshold it locates

B. Stage One: The Baseline Distribution

The first stage of the solution concerns projects that no policy should keep. A project earning less than $(R_f - 1)(1 - \gamma)$ is dominated even after tax: the investor can distribute the dollar, pay the tax, reinvest at the risk-free rate, and still come out ahead. The hurdle is the retained-earnings hurdle of Section II: the ordering rule puts retained earnings first in line, so the first distributed dollars bear the tax. Equity composition itself enters at the second stage, once the Baseline Distribution has been paid. To isolate the access margin visually, the calibration of Table 2 and Figure 1 sets the Baseline Distribution to zero.

Proposition 1 (Baseline Distribution). Let a, $\gamma > 0$ and δ, $\sigma \geqslant 0$, let $0 < RE < BV$, and let all functions and definitions be as in the setup. Then every distribution $d \geq 0$ with $d < d_{RE}$

satisfies $TR(d) < TR(d_{RE})$: any policy that distributes less than the Baseline Distribution is strictly dominated. Proved in Appendix B, Part A, where the existence and uniqueness of d_{RE} is Lemma 1.

In words: some projects should not survive under any policy. A project earning less than what a distributed dollar earns outside the firm, even after paying the distribution tax on the way out, is dominated regardless of what else the firm decides. Every optimal policy therefore begins by shedding these projects, and the interesting decision, taken up next, concerns only the capital that clears this after-tax floor.

C. Stage Two: The Access Decision

After the baseline, the question is whether to distribute more, and here the ordering rule and the tax combine to make the choice all-or-nothing. Every additional dollar comes from retained earnings and is taxed, while the projects still inside all earn more than the after-tax hurdle, the baseline having removed those below it, so pushing them out through the taxed layer is a strict loss dollar by dollar. Yet those earning between $(\mathrm{Rf} - 1)(1 - \gamma)$ and $\mathrm{Rf} - 1$ would be worth shedding if they could leave untaxed, and the only way to shed them untaxed is to reach the contributed-capital layer, which the rule places last. The resulting policy is binary, and everything downstream follows from it.

Proposition 2 (Discreteness). Let $a, \delta > 0$, $0 < \gamma < 1$, $0 < RE < BV$, and $\sigma = 0$, with all functions and definitions as in the setup. Then the set of optimal distributions D^* is non-empty, and $D^* \subseteq \{0, d_C\}$: beyond the baseline the firm either distributes nothing or undertakes the Access Distribution, and no other distribution level is optimal. Proved in Appendix B, Part B, where the existence and uniqueness of d_C is Lemma 2.

In words: the choice beyond the baseline is indivisible. An intermediate distribution pays the toll of pushing dollars through the taxed retained-earnings layer without reaching the tax-free contributed capital where the remaining gains sit: all of the cost, none of the prize. So the firm stops or goes all the way, and the two regimes, the kinked returns, and the kinked value all inherit their geometry from that structure. The discreteness requires both institutional features at once: the ordering rule layers the exit and differential taxation makes the layers differ, and removing either brings the interior back. It does not require the parametric schedule, as the following remark records.

Remark (Generality). Neither the power form nor the normalization $\sigma = 0$ carries the structure of the solution. For any continuous, strictly increasing schedule whose range spans the two hurdles, and with the baseline drawn from retained earnings as above, the optimum still sits at one of the two candidate policies and never between them.[16] The power form and $\sigma = 0$ are needed only for the closed forms of Proposition 3. Nothing else in the analysis depends on them.

What remains is the condition for access: the firm accesses when the gains from divesting the remaining below-market projects exceed the toll of first distributing all retained earnings. Proposition 3 states it exactly, under the parametric schedule.

Proposition 3 (Optimal Distribution). Let a, $\delta > 0$, $0 < \gamma < 1$, $0 < RE < BV$, and $\sigma = 0$, with all functions and definitions as in the setup and $\theta = \gamma(R_f - 1)/[\gamma(R_f - 1) + \delta]$. Under the power-function schedule, the Access Distribution is $d_C = BV\theta^{1/a}$. Access is optimal if and only if $RE/BV \leqslant a\ \theta^{1/a}/(a + 1)$, and retention if and only if $RE/BV \geqslant a\ \theta^{1/a}/(a + 1)$, so both are optimal,

[16] Let f be continuous and strictly increasing on $[0, BV]$ with $f(0) \leqslant (R_f - 1)(1 - \gamma)$ and $f(BV) \geqslant R_f - 1$, let g be as in the setup, and let $d_{RE} \leqslant RE$. Both benchmark distributions exist and are unique, by the intermediate value theorem and strict monotonicity. The marginal value of distributing the next dollar is $g(d) - f(d)$. Below d_{RE} the dollar earns more outside than inside, so total returns rise; on (d_{RE}, RE) the kept dollar beats its taxed exit, so they fall; on (RE, d_C) the exit is untaxed while the projects still inside earn less than the market, so they rise again; beyond d_C they fall. Total returns are continuous in d, so they peak at d_{RE}, sag, and recover only at d_C, and the optimum is at d_{RE} or at d_C. Under $\sigma = 0$ these are the policies of Proposition 2, since $d_{RE} = 0$ there. When d_C does not exceed RE, total returns fall everywhere past d_{RE} and retention is optimal outright.

and the investor indifferent between them, exactly at the threshold $RE^* = BV \bullet a\ \theta^{1/a}/(a + 1)$. Proved in Appendix B, Part B.[17]

In words: access is chosen when the prize beats the toll, and the condition moves as intuition suggests. Access is less likely when retained earnings are large relative to book value, since the toll is larger for the same prize; less likely when internal opportunities are strong, since divestiture then forgoes more; and more likely when projects cluster at low returns, since the prize from shedding them is larger.

Two different things move here, and it helps to keep them apart. Equity composition sets which side of the threshold a firm falls on and nothing more, since it does not enter d_C. The return schedule sets both where the threshold sits and how large the Access Distribution is, so a firm with weaker internal opportunities faces a higher threshold and sheds more when it does access. The all-or-nothing structure that makes the threshold matter at all is Proposition 2's.

D. The Two Regimes

The binary policy sorts firms into two regimes, and the labels carry through the paper. A firm in the retention regime pays the baseline and stops, its retained earnings staying inside; a firm in the access regime pays the toll, clears them, and reaches its contributed capital. Two firms identical but for composition can therefore choose differently, since the one financed mostly by retained earnings faces a larger toll for the same prize. Composition determines policy, policy determines which projects the firm keeps, and the kept projects determine expected earnings. That is the chain of Section III with a discrete policy in place of the smooth margin, its

[17] Setting $\sigma = 0$ is *a* convenience for the closed forms rather than *a* load-bearing restriction, as the Remark following Proposition 2 records. Under $\sigma > 0$ the Access Distribution keeps the same form with θ replaced by $[\gamma(R_f - 1) + \sigma]/[\gamma(R_f - 1) + \delta + \sigma]$, while the threshold acquires additional σ terms. Because the normalization anchors the weakest project to the after-tax hurdle, the closed forms describe *a* given tax regime, and comparative statics in the tax rate should hold the schedule fixed, as in Section VI.

second link now resting on Propositions 1 through 3 rather than on the per-dollar comparison of Section II.B.

A numerical illustration makes the sorting concrete. Suppose the risk-free rate is 4 percent and the distribution-tax rate is 25 percent, so a distributed retained-earnings dollar returns 3 percent after tax while a returned contributed-capital dollar earns 4 percent, and internal project returns range from 2 to 6 percent. Firm A holds $RE/BV = 0.80$ and Firm B holds $RE/BV = 0.20$. Both pay the baseline, divesting every project earning below 3 percent. Reaching contributed capital, however, requires first distributing all retained earnings: for Firm B that toll is small, so it undertakes the Access Distribution, sheds its 3 to 4 percent projects as well, and shareholders reinvest the proceeds externally; for Firm A the toll is large enough to make the same reallocation unattractive, and it remains in retention. Composition alone separates them.

The access margin is therefore populated less by mature dividend payers, whose large retained earnings hold them in retention, than by firms whose distributions approach paid-in capital through leveraged recapitalizations, special dividends, or other deep return-of-capital transactions. That does not make this a theory only of low-retained-earnings firms. The retention regime is itself a prediction: firms with substantial retained earnings may forgo otherwise attractive reallocations because crossing the taxable layer costs too much. The access threshold is simply where the mechanism becomes most visible.

The section has established what equity composition does to policy. It does not shift a continuous margin; it selects between two policies, and the selection turns on where the firm sits relative to a threshold set by its return schedule and the tax rate. What that discreteness does to returns, and then to value, is the subject of Section VI.

V. DYNAMICS, DISTRIBUTION FUNDING, AND REPURCHASES

Three objections stand between the policy result and the geometry that follows. Each, if valid, would break the link between them. The first is that the model is static. The second is that firms fund distributions out of cash rather than by selling projects. The third is that firms can repurchase rather than pay dividends. This section answers them in turn.

A. Scope and Dynamics

The analysis is local. It characterizes policy at a point in time, and through Section VI the valuation geometry at that point, rather than a full dynamic payout path. The results are comparative statics. They compare otherwise-identical firms that differ in equity composition at a point in time, the same construction as the two-firm comparison above, so no law of motion for the state is required. Two named assumptions set the scope. The first is Eventual Taxation, carried from Section II and discharged in Section IV.A. The second concerns persistence: when Section VI converts returns to value, the post-distribution allocation is held fixed and the composition-induced component of abnormal earnings is treated as non-decaying, $\omega = 1$, because it reflects the return differential that a persistent tax wedge generates from a fixed allocation, rather than a competitive rent that rivalry erodes. Section VI.C states this two-component treatment and the alternative case in which the component decays.

The discreteness is not an artifact of the single period. The ordering rule poses the same indivisible choice in every period it applies, and because the tax rate does not depend on when the distribution occurs, deferral changes the timing of the toll rather than its structure, so it cannot convert the per-period discreteness into a smooth marginal adjustment. Dynamic optimization would move the threshold through the option value of waiting and could attenuate the equity-composition effect. The static magnitudes derived here should therefore be read as local benchmark effects rather than full dynamic magnitudes. Modeling the timing of access is left to companion work.

B. The Source of Distributed Funds

A reader may object that firms often fund distributions from cash rather than by selling operating projects. The friction, however, is the tax ordering of the distribution, not the physical source of funds. Returning contributed capital requires first crossing the retained-earnings layer, whether the cash comes from asset sales, financial slack, or accumulated cash balances. In the model, cash and financial slack can be represented as points on the same return schedule, with cash earning the risk-free rate. Thus the divestiture formalization should be read as a reduced-form way to model the assets or slack displaced by a distribution, not as a literal claim that every payout requires selling operating projects.

C. Repurchases

The objection deserves its strongest form. Repurchases carry much of corporate payout, their tax turns on a shareholder's basis rather than on the firm's split between retained earnings and contributed capital, and aggregate shareholder basis can diverge substantially from contributed capital as shares change hands. In its sharpest form the objection is not about magnitude at all: a firm that distributes only through repurchases never reaches the earnings-and-profits ordering rule, so the gate this paper builds on is never opened, and equity composition would appear to have nothing left to govern.

Where value is eventually realized in taxable hands, repurchases change who bears the shareholder-level tax and when; they do not change how much of the firm's value is taxed. Since 2003, qualified dividends and long-term gains face essentially the same rate.[18] With the rates

[18] Since 2023, IRC §4501 adds a firm-level excise tax of 1 percent on the fair market value of net repurchases (Inflation Reduction Act of 2022; final regulations issued November 2025), narrowing the remaining tax advantage of the repurchase channel. The excise tax operates at the firm level and leaves

equal, what a dividend and a repurchase tax differently is the base of a single transaction: a dividend taxes the whole distribution, while a repurchase taxes only the gain and returns basis free of tax. Across transactions the difference disappears. A shareholder is taxed on what he did not pay for, recovering what he paid free of tax and bearing tax on the rest, and summed over the chain of holders from issuance to the final return of capital, those taxed amounts collapse to the value that left the firm less the capital paid into it. Shareholders as a group are therefore eventually taxed on everything they did not pay in, and what they paid in is contributed capital. Nothing in that sum depends on how often shares change hands, on the prices at which they trade, or on the mix of dividends and repurchases the firm chooses. The reason is that whatever a repurchase leaves untaxed is basis, and basis is either capital originally paid in or an amount a predecessor was already taxed on, so the exemption never adds to the untaxed total.[19]

The identity settles the question the objection raised, whether the choice of instrument can shrink the tax on retained earnings, and it settles only that question. The gate the objection said would never open does not need to open, because the toll is collected through whichever channel finally carries the value out. Retained earnings cannot be moved out of the firm untaxed by a change of instrument, so the taxation half of Eventual Taxation is a result rather than an assumption. What remains assumed is the other half, that value eventually leaves the firm. What lies outside the identity is timing.

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the shareholder-level basis logic of this subsection unchanged; empirically, its introduction reduced repurchases without a dividend offset (Bargeron et al., 2024; Autore, Barnes, Clarke, & Schrowang, 2025).

[19] A firm with book value of 100 and contributed capital of 20 illustrates the invariance. Let the original holder sell his shares at any price P and the buyer later be bought out for 100. The seller is taxed on P less 20 and the buyer on 100 less P, which sum to 80 whatever P may be. If instead the firm distributes 80 as a dividend before the sale, the original holder is taxed on 80 and no gain remains for anyone else. The taxed amounts sum to the retained earnings in either case, and the same holds for any sequence of trades and distributions in between.

The timing that matters is the shareholder's, not the firm's. Deferring the distribution itself is a wash: the base grows while the rate stays constant, the same logic by which the new view makes dividend timing neutral. Deferring realization is not a wash, because basis is fixed in nominal dollars while value compounds, so a holder who waits earns returns on the government's share in the interim. That compression of present value, and only that, is what the blended rate γ carries. Reaching today's decision from a lifetime base takes one further step, the ordinary one that prices reflect anticipated shareholder-level taxes, as they do throughout Sections II and III.[20] For the current-period decision the consequence is direct: a dollar distributed from the retained-earnings layer bears the toll at the effective rate whichever instrument carries it, so the after-tax hurdle that sets policy in Section IV stands, and equity composition, which sets the base, governs the distribution decision.

Three limits bound the claim. Value must eventually leave the firm, which the model assumes rather than derives. The chain must end in taxable hands, since a step-up at death or a transfer to an exempt holder extinguishes the final term, so for a clientele holding until death the wedge does not merely compress, it closes. And deferral compresses the present value of a toll the identity fixes only in total. γ is accordingly a blended effective rate across payout channels, basis positions, clienteles, and realization horizons rather than the statutory dividend rate. Appendix A.E states the identity formally over a complete history, supplies the statutory grounding, develops that rate, and documents that repurchases have not displaced the dividend channel the ordering rule governs.

None of the three objections that opened this section, the static structure, the source of distributed funds, and repurchases, removes the discreteness. The single-period structure poses

[20] That prices reflect anticipated shareholder-level taxes is a modeling convention, carried throughout Sections II and III. It is distinct from the empirical claim that the dividend tax is capitalized into observed stock prices, which remains contested and on which nothing in this paper depends.

the same indivisible choice in every period it applies, the tax ordering binds whatever the physical source of the funds, and repurchases retime the toll without shrinking the base it is charged on. The geometry derived next therefore rests on a policy result that survives all three.

VI. THE GEOMETRY OF RETURNS AND VALUATION

Section IV established the policy: under the ordering rule, optimal payout is binary, and equity composition selects the regime. Two questions remain. What does the discrete policy do to the returns investors earn, and what does it do to the price of the firm? This section answers both in turn. Three terms recur along the way, and they are not stylistic variants: the kink is the change in slope, the corner is the shape that change takes when the threshold is known, and the bend is the smoothed shape it takes when the threshold is uncertain. Appendix B contains the proofs.

A. The Kink in Total Returns

Section IV's two otherwise identical firms diverge in their returns. Holding the same schedule of projects in place, they earn different total returns to investors, a dollar flow of after-tax returns rather than a rate, and different average internal rates of return on the assets that remain inside the firm, solely because equity composition sends them to opposite distribution decisions. The comparison concerns the divestiture of assets in place rather than the selection of new projects, and it is cross-regime: it requires a composition gap wide enough to change the policy each firm chooses. Within a regime composition works at the margin, which is Proposition 5 below.

Proposition 4 (Regime Returns). Let two firms, A and B, face the same return schedule, with the schedule curvature a and the parameters γ, δ, and σ identical across firms, where $a, \delta > 0$, $0 < \gamma < 1$, and $\sigma = 0$, and all functions and definitions as in the setup. Let the firms have equal book value, $BV_A = BV_B = BV$, and let firm A hold more retained earnings, $0 < RE_B < RE_A < BV$.

If the two firms' optimal distribution policies differ, so that neither firm's optimal set contains the other's, then firm B earns strictly higher optimized total returns, $TR_B^* > TR_A^*$, and a strictly higher average internal rate of return on the funds remaining inside it, $AR_B^* > AR_A^*$. Proved in Appendix B, Part C.

In words: two firms identical but for the composition of their equity can deliver different returns to their investors and run different-quality books of projects. The contributed-capital firm pays a smaller toll to reach its tax-free layer, so it sheds the projects at the bottom of its schedule and the proceeds earn the external return in investors' hands; the retained-earnings firm keeps them.[21] Both halves of the result carry weight later. The total-returns half feeds the valuation geometry of the next subsection, and the internal-returns half, the jump in the average quality of what stays inside the firm, is what the investment tests of Section VII measure.

How do total returns behave as composition crosses the threshold? Proposition 5 answers in terms of the threshold ratio θ of Section IV.A, the tax toll on a retained-earnings dollar as a share of that toll plus the premium δ.

Proposition 5 (Kinked Returns). Let all functions, definitions, and assumptions be as in Proposition 4, and define $TR^*(RE)$ as the total returns realized by the investor under the optimal distribution choice, conditional on the firm's pre-distribution retained earnings, holding $BV > 0$ and the return schedule fixed. Then $TR^*(RE)$ is continuous over its domain $[0, BV]$ and kinked at the threshold $RE^* = BV \cdot \mathrm{a}\theta^{1/a}/(a + 1)$: the one-sided derivative of TR^* with respect to RE is $-\gamma(R_f - 1)$ on the access side of the threshold and zero on the retention side, so the slope change at the threshold is $\mathrm{K}_{TR} = \gamma(R_f - 1)$, a function of the distribution-tax rate and the risk-free rate alone,

[21] The pattern has an empirical counterpart in the payout-tax literature, which finds that high payout taxes lock investment into profitable, internally financed firms and so alter the allocation of capital across firms (Becker, Jacob, & Jacob, 2013; Alstadsæter, Jacob, & Michaely, 2017). That work identifies the margin between internal and external funding of new investment, whereas the margin here is the divestiture of assets already in place.

with the schedule parameters entering only through the location of RE^*. Proved in Appendix B, Part C.

In words: returns are continuous at the threshold, because there the firm is indifferent between its two policies, but the marginal price of composition changes discretely. On the retention side a reclassified dollar changes nothing the firm does, so it changes nothing the investor earns. On the access side each additional dollar of retained earnings enlarges the taxed layer the distribution must cross, and it costs the investor exactly the tax on one distributed dollar's return. The sharpness of the corner depends only on the tax rate and the risk-free rate; where it sits depends on the firm's schedule.

The reason the schedule parameters drop out is that, by Proposition 3, the size of the Access Distribution is set by book value and the return schedule and retained earnings do not enter it: composition decides only whether the firm makes that distribution, never how large it is, and changes only how much of the distributed amount bears the tax.

The result also separates what jumps at the regime boundary from what does not. The firm's behavior jumps: it goes from making no Access Distribution at all to distributing the amount that clears its retained earnings and reaches its contributed capital, and the average return on the projects it keeps jumps with that divestiture. What investors receive does not jump. Being at the threshold means the firm is exactly indifferent between the two policies, so both deliver the same total returns there and, as shown next, the same value. The discontinuity is in the policy and in the slope, not in the level of returns or of value.

B. The Valuation Kink

The results so far are statements about returns. Converting them into price takes one further step, though no new valuation framework: equation (9) prices the market value of equity as tax-adjusted book value plus capitalized abnormal earnings, and the optimized flow of Section

VI.A supplies those abnormal earnings. The accounting articulates throughout, since retained earnings and contributed capital evolve by the identities of Section II, so book value and the flows satisfy clean surplus and value is measured to shareholders after personal tax. Holding the post-distribution allocation fixed, firm value as a function of equity composition is

$$FV(RE) = (BV - \gamma RE) + \alpha_1(TR^*(RE) - G(BV, RE)),$$

tax-adjusted book value plus persistence-weighted abnormal earnings, where $TR^*(RE)$ is the optimized after-tax return flow of Proposition 5 and $G(BV, RE)$, the benchmark charge, is the flow the same book equity would earn if distributed in full and reinvested at the after-tax external returns.[22] Appendix B, Part D, derives this in all three cases the optimal policy admits and shows it is equation (9) applied to the optimized flow rather than an alternative to it. Because Section III's linear model is itself derived from the present value of after-tax distributions in Section II, discounting the optimized flows directly returns the same value, so the kink is a property of the flows the policy generates rather than of the representation used to display them.

The valuation is cum-distribution: it prices the firm once the optimal policy is set and before the distribution is paid, as a stock is priced after a dividend is declared and before the ex-date.

One identity locates the whole geometry inside equation (9). Evaluating the benchmark charge at full book value gives

$$\begin{aligned} G(BV, RE) &= (R_f - 1)(1 - \gamma)RE + (R_f - 1)(BV - RE) \\ &= (R_f - 1)(BV - \gamma RE) = (R_f - 1)[C + (1 - \gamma)RE]. \end{aligned}$$

$G(BV, RE)$ is exactly the normal-return charge of equation (9): the after-tax normal return on tax-adjusted book value, with retained earnings entering at the taxed external return and

[22] Equation (9) charges the normal return on lagged book value, while G(BV, RE) charges it on current book value. The comparison holds the post-distribution allocation fixed period to period, so lagged and current book coincide and the two charges are the same object.

contributed capital at the untaxed one. Nothing needs grossing up, and nothing is divided by $(1 - \gamma)$.

The two mechanisms of the paper divide cleanly here. Differential taxation creates the wedge: it is what makes the benchmark tax-adjusted book value rather than book value, as Section II derives before any sequencing constraint. The ordering rule does not create that adjustment. What it does is guarantee that the adjustment is borne, by converting Eventual Taxation from assumption to property of the exit, and it then governs how the firm's optimized flow departs from the benchmark as equity composition changes. That is why abnormal earnings here are measured against the composite normal return of Section II rather than a uniform cost of capital.

One clarification forestalls a misreading. Nothing assigns particular projects to retained earnings or to contributed capital; assets are fungible with respect to their financing labels, and the charge does not claim that each component earns its own hurdle rate. The component rates are outside options. Contributed capital would support the full external return if it could be reached, but while it sits behind retained earnings the firm may rationally keep projects earning between the after-tax hurdle and that benchmark. The resulting drag on earnings is what the valuation prices rather than assumes away.

Proposition 6 (Kinked Value). Define firm value, as a function of retained earnings holding book value fixed, as $FV(RE) = BV - \gamma RE + \alpha_1(TR^*(RE) - \mathrm{G}(BV, RE))$, with all functions and definitions as in the setup and $\alpha_1 = \omega/(R_f - \omega)$ as in equation (9). Then $FV(RE)$ is continuous in RE over $[0, BV]$ and kinked at the interior threshold $RE^* = BV \cdot \mathrm{a}\theta^{1/a}/(a + 1)$: the one-sided slope is $\alpha_1\gamma(R_f - 1) - \gamma$ on the retention side and $-\gamma$ on the access side, so the slope change at the threshold is $\mathrm{K}_{FV} = \alpha_1\mathrm{K}_{TR} = \alpha_1\gamma(R_f - 1)$. At $\omega = 1$ the retention-side slope is zero and the kink is exactly γ. The domain is closed because FV extends continuously to $RE = 0$ and $RE = BV$, where the two-regime policy structure is degenerate but value remains well defined; Propositions 1 through 3 restrict attention to the interior. Proved in Appendix B, Part D.

In words: value is tax-adjusted book value plus capitalized abnormal earnings. Recall the offset that made composition irrelevant in the benchmark, in Sections II.B and III.C: the tax discount applied to retained earnings inside the book term was cancelled by the lower normal-return charge that same principal has to cover, so composition washed out of value. The kink is the point where that cancellation fails. The book term falls by the tax rate for every dollar of retained earnings in both regimes; that is what tax adjustment means. In the retention regime policy does not move with composition, so the optimized flow is unchanged while the benchmark falls and abnormal earnings rise, offsetting part of the decline in the book term and all of it when the composition-induced component is stationary. In the access regime the firm is reallocating, the optimized flow falls in step with the benchmark, and the book term's slope passes through to value. The change is then the tax rate, because that is what the book term carries. Figure 1 draws the two regimes and the corner between them.

Linear capitalization fixes the magnitude of the kink, not its existence. Under any pricing of the optimized flow that is strictly increasing and differentiable at the threshold, the slope change is scaled by that function's derivative, so the threshold stays put and a corner survives wherever the derivative is positive. Linearity is what pins the change at exactly γ.

C. Persistence

One question is left over from Proposition 6: is the kink really the tax rate, or does that depend on how long abnormal earnings last? The corner survives either way, and only its size turns on persistence. Settling the size takes one extension, stated here rather than slipped in. Proposition 6 applies a single Ohlson persistence parameter to the whole abnormal-earnings term; for the case drawn in Table 2 and Figure 1, we split that term into the composition-varying flow of Section VI.B and a composition-invariant remainder, and we let the two carry different persistence.

The composition-varying flow is the return differential a persistent tax wedge generates from an allocation the comparison holds fixed, so the main case takes $\omega = 1$ for it and capitalizes it at $1/(Rf - 1)$. That is Ohlson's stationary limit applied to one component rather than a claim that abnormal earnings are permanent, and it prices the composition the firm has now, the local comparative static of Section V.A. The remainder, competitive rents plus the other information ν, keeps its ordinary persistence and is capitalized at $\alpha 1$ and $\alpha 2$ as in equation (9); because it is common to firms identical but for composition, it differences out of every slope and out of the kink. Value then reads

$$FV(RE) = (BV - \gamma RE) + A_c(RE)/(R_f - 1) + \alpha_1 A_0 + \alpha_2 \nu,$$

where Ac, normalized to vanish at the threshold, is the composition-varying component and A0 the remainder. The slopes are zero in the retention regime and $-\gamma$ in the access regime, and the kink is exactly γ: the slopes Proposition 6 delivers at $\omega = 1$, reached without imposing that persistence on the remainder. Table 2 and Figure 1 report them. Across the three calibrations of Table 2 the schedule moves the threshold from roughly 8 to 53 percent of book value while the kink stays at 0.20, so the schedule decides where the corner sits, never how sharp it is.[23]

Remark (imperfect persistence). Proposition 6 is general in ω, so if the composition-induced flow itself decays, every component is capitalized at $\alpha 1$ and the kink scales to $KFV = \alpha 1\gamma(Rf - 1) = \alpha 1KTR$, approaching γ as ω approaches one. The bend is then sharper where

[23] Two magnitudes coincide here without being the same object: because the retention-regime slope is zero, the access-regime slope and the kink both come out at the tax rate, one being the slope below the threshold and the other the change in slope at it. The exactness of the retention-side offset rests on Eventual Taxation, which the ordering rule converts into a property of the exit (Section IV); under only partial eventual taxation the offset would be incomplete and the kink would scale down with it. Empirical estimates of ω, roughly 0.6 at annual horizons, are fitted to aggregate abnormal earnings dominated by competitive rents, and the paper applies them to the remainder rather than to the composition-induced component.

abnormal earnings persist, a comparative static restated beside Prediction A in Section VII. Nothing in Appendix B changes between the two cases.

D. Magnitude

The magnitude can be stated in dollars. Qualified dividends in the United States are taxed at the long-term capital gains rates of 0, 15, or 20 percent, and the 3.8 percent net investment income tax applies above its income thresholds. For a taxable investor subject to that tax, the combined rate is 18.8 or 23.8 percent, which gives upper-end calibration points for γ. Because γ is a clientele-weighted effective rate blending payout channel, basis, horizon, and investor type, the rate that prices the firm will generally be lower.

At those rates the predicted slope change at the threshold is roughly 19 to 24 cents of firm value per dollar of retained earnings, with the $\gamma = 0.20$ calibration of Figure 1 in the middle of the band. Against the usual benchmark in which book equity carries a valuation coefficient near one, a slope difference of about twenty cents on the equity-composition margin is first-order, not a curiosity.

E. The Kink under Uncertainty

The kink so far presumes investors observe the threshold exactly. For the uncertain case, write $V(RE; RE^*)$ for the value function at a known threshold RE^*, taken in the main case of Section VI.C, where the retention-side slope is zero and the access-side slope is $-\gamma$; Appendix B, Part E, records its kinked form and proves the following. Under general persistence the same argument applies with the two regime slopes of Proposition 6 in place of 0 and $-\gamma$.

Corollary 1 (Kink under Uncertainty). Suppose the threshold RE^* is uncertain, with continuous distribution function F over a bounded support, the firm's other fundamentals held fixed. Then expected value $E[V(RE;\ RE^*)]$ is continuously differentiable in RE with slope $-\gamma(1 - F(RE))$: a smooth, convex function whose slope rises from $-\gamma$ below the support to 0 above it.

The corner survives in expectation as curvature, and the integrated slope change across the support is exactly γ.

In words: when investors cannot pinpoint where the firm's threshold lies, the corner is not erased but spread out. Value bends smoothly across the range of possible thresholds, and the total slope change across that range is still the tax rate, so uncertainty changes the shape of the relation without changing its size.

The reason is that the kink is the certainty form of the relation. Firm value is the upper envelope of the retention and access valuations, two lines whose slopes differ by γ and that meet at the threshold; under certainty the firm knows which side of the threshold it occupies, and the relation has a corner. When the threshold is uncertain, because the return schedule or its parameters are not known in advance, the priced relation is the expectation of that upper envelope, and the expectation of a maximum of two lines is smooth and convex. Where effective rates differ across investors, the curvature scales with the relevant blend of those rates rather than with a single γ.

The smoothing falls on value alone. By Proposition 2 the firm still pays either the baseline or the access distribution in every realized state, so payouts and divestitures remain lumpy while only the priced relation bends. Taken across firms rather than within one, the same expectation produces the pooled cross-sectional bend of Section VII. The proof and its resolution notes are in Appendix B, Part E.

F. Relation to Prior Results

Two questions organize the prior results considered here: does the distribution ordering rule bind, and does capital allocation respond to equity composition? They define a two-by-two, and Table 1 places the established results in it. Three cells are occupied by prior work; the fourth, where the rule binds and allocation is endogenous, is where this paper sits. The four are not equally realistic. The ordering rule is a standing feature of U.S. law for ordinary distributions and firms do reallocate capital, so the fourth cell describes the world as it stands while the other three are benchmarks reached by switching off one real feature at a time. That is their value: each prior result identifies exactly which counterfactual generates it, the fixed-allocation cells following from equation (11) and the no-rule cell from the remark below.

Table 1. Nested benchmarks: the ordering rule and allocation endogeneity.

	Allocation fixed (earnings exogenous)	**Allocation endogenous**
Ordering rule absent	Composition irrelevance: $\partial P/\partial RE = 0$. Miller and Modigliani (1961); Hanlon, Myers, and Shevlin (2003)	Smooth cost-of-capital effect: value smooth in *RE/BV*, no kink at RE^*. Auerbach (1979); Sinn (1991); Zodrow (1991)
Ordering rule present	Composition irrelevance: $\partial P/\partial RE = 0$. The ordering rule is inert while allocation is fixed; the tax-adjusted discount is offset in valuation. Guenther and Sansing (2006); Sections II–III	**This paper (Sections IV–VI):** piecewise linear, convex kink γ at the value threshold RE^*

Remark (benchmark without the ordering rule). Remove the rule and let the firm designate each distributed dollar as drawn from retained earnings or from contributed capital at will. Designating contributed capital first weakly dominates, since it changes only the tax on the distributed dollar, γ against zero, and leaves the allocation unchanged. The bands of the distributed-return function interchange, so a distributed dollar earns $Rf - 1$ up to C and $(Rf - 1)(1 - \gamma)$ beyond it. The firm then divests every project earning below $Rf - 1$, a boundary fixed by the schedule and independent of equity composition, and it funds the divestiture tax-free whenever it does not exceed contributed capital. No distribution tax is paid along the optimal path, the toll that generates the discrete access decision never arises, and value is locally independent of composition. Composition binds only when the desired divestiture exceeds C, and that boundary is a different locus from RE* and does not carry the γ magnitude. The kink is therefore a product of the ordering rule and not of differential taxation alone.

The result stands in a definite relation to dividend-tax neutrality, and it is one of scope rather than contradiction. Neutrality concerns the retain-and-invest margin: a tax borne on any eventual distribution does not distort the choice to retain and invest internally, since it falls equally on distributing now or later (King, 1977; Auerbach & Hassett, 2002). The ordering rule creates a second margin. Returning contributed capital requires routing every retained-earnings dollar through the distribution tax first, so the tax that is neutral for retain-and-invest becomes a binding cost at the return-of-capital margin. The kink is the price of that access. By Section V.C the instrument cannot avoid that price, though it changes who bears it and when, so the effective rate rather than the statutory one is what the geometry carries.

As a comparative static, the distinction separates the mechanism from both standard views. Hold the return schedule fixed and cut the rate: the toll on reaching contributed capital falls while the tax-free amount is unchanged, so deep return-of-capital distributions rise, while investment financed at the internal return moves only within the narrow band the rate change opens. Neither standard view generates that joint pattern.[24]

G. What the Kink Means

Three implications follow. First, equity composition and book value together determine which side of the threshold the firm occupies, and that is what aggregated book equity conceals: two firms reporting the same book equity can sit in different regimes, and price depends on which. Second, expected earnings are no longer an exogenous input. They are produced by the firm's optimal distribution and reinvestment policy, which composition itself selects. Third, the

[24] A direct analogue is the taxation of repatriated foreign earnings, where a tax on returning capital led firms to hold earnings abroad rather than pay it out, an avoidance response rather than a change in investment (Foley, Hartzell, Titman, & Twite, 2007).

global linear relation of Section III does not survive that endogeneity. For a firm at the distribution margin, value takes the piecewise-linear, convex form derived above, with the threshold characterized in Proposition 3. Equity composition is in that sense a state variable governing the geometry of valuation.

Standard abnormal-earnings models treat book value as a sufficient statistic for expected normal earnings and so abstract from this structure entirely. Once personal taxation and the ordering rule enter together, the retained-earnings share determines not only the level of value but the functional form of the relation.

VII. OBSERVABLE IMPLICATIONS OF THE MECHANISM

Unusually for a valuation model, the central implication concerns a magnitude and not only a sign: the slope change at the threshold equals the effective distribution-tax rate, not a coefficient to be freely fitted. One mechanism produces several predictions, and each is stated below with the result it comes from and what would reject it.

Two caveats apply throughout. Equity composition correlates with maturity, and risk correlated with RE/BV can contaminate estimated slopes through discount rates, so every prediction below holds risk fixed. And book retained earnings stands throughout for tax earnings and profits, the variable the rule actually operates on, with the proxy error concentrated near the access threshold; Appendix A gives the implementation guidance.

A. The Convex Bend in Value

Proposition 6 gives the corner and Corollary 1 rounds it into a bend. The rounding happens twice over: within a firm, because the threshold depends on a return schedule investors do not observe exactly, and across firms, because thresholds differ. Both are the same expectation of an upper envelope, which is why the same γ scales the curvature in each.

Prediction A (Convex Bend in Value). For firms at the distribution margin, holding risk fixed, value is convex in *RE/BV*: the slope is near zero at high *RE/BV* and approaches $-\gamma$ at low *RE/BV*, with the curvature concentrated near the firm's return-of-capital margin rather than spread across the equity-composition support. The observable content is the location and the scaling of that curvature. It sharpens with reliance on dividends and with the depth of access, since shallow repurchases defer most of the toll while deep access brings it forward; it attenuates as ownership shifts toward tax-exempt hands; and within jurisdiction it attenuates after the 2003 dividend-tax cut lowered γ, for firms held by taxable rather than tax-exempt investors (Appendix A.E). Section VI states the dollar magnitude. A smooth life-cycle decline in valuation multiples can also produce curvature in *RE/BV*, but it has no reason to share these tax-specific comparative statics or the localization at the return-of-capital margin; the falsifier is curvature that fails to scale with tax-rate proxies or that spreads evenly across the support.

A companion comparative static follows from the alternative case of Section VI.C: if the composition-induced flow itself decays, the kink scales as $\alpha 1\gamma(Rf - 1)$, so the bend is sharper where abnormal earnings persist. The prediction is directional rather than sized.

Earlier work documents level associations between retained earnings and value (Harris & Kemsley, 1999), evidence whose interpretation remains contested. The model neither requires nor predicts those level effects, since composition irrelevance still holds wherever expected earnings are held fixed. What it supplies is a home for the interaction those studies estimated: the composition effect concentrates where distributions approach the return-of-capital margin.

B. Lumpy Payout at Depth

Proposition 2 rules out interior distributions and Proposition 3 says when access is chosen: when retained earnings are small relative to book value and internal returns cluster low.

So the incidence of deep distributions should move with equity composition, not merely with free cash flow.

Prediction B (Lumpy Payout at Depth). Distributions that reach paid-in capital arrive as large, discrete events, such as leveraged recapitalizations, special dividends, and other deep return-of-capital transactions, and their incidence concentrates at low *RE/BV*; distributions sized between the baseline and full access are rare. The model permits ordinary recurring dividends, so lumpiness is a statement about the incremental access decision, not about all payout. The payout literature documents lumpiness and the flexibility value of repurchases (Skinner, 2008); what this framework adds, and what only this framework predicts, is that the lumpiness concentrates at specific values of equity composition, because composition sets the toll. The falsifier is a population of interior-sized distributions at the margin, or deep-access incidence unrelated to equity composition.

C. Regime-Dependent Investment and Earnings

Proposition 4 works through what each policy divests. The Baseline Distribution removes only projects below the after-tax hurdle, while the Access Distribution also removes the band between the two hurdles, so crossing into access sheds the firm's weakest remaining projects.

Prediction C (Regime-Dependent Investment and Earnings). Average internal returns on continuing operations, AR^* in the model and measured as Tobin's Q or ROIC on the retained asset base, jump upward at deep-access events. The prediction is cross-regime: under the ordering rule the allocation is fixed within a regime, so crossing into access is what raises the average quality of the assets remaining inside the firm. The smooth counterpart belongs to the benchmark of Section III, where before the rule is imposed a higher RE/BV lowers the composite normal return of equation (12) and so lowers expected normal earnings through earnings generation rather than discount rates. That smooth channel claims a pathway:

composition moves value through expected earnings, so controlling for realized future earnings should absorb the RE/BV coefficient, and a coefficient that survives at full strength rejects the pathway. The investment falsifier is simpler, the absence of any jump in internal-return measures at access events.

D. The Joint Signature

At bottom the three are one prediction: equity composition sets the regime, and the regime moves payout, investment quality, and the valuation slope together at the same boundary. That coincidence is what a linear test cannot see.

Prediction D (Joint Signature). The diagnostic that linear tests miss is joint: a deep-access event should coincide with a discrete payout, an upward jump in average internal returns, and a local steepening of the value relation, all at the same boundary. The valuation slope change is the one scaled by γ; the payout and investment responses move with γ through the location of the access threshold rather than in proportion to it. Any one signature has confounds, and no single alternative explanation readily generates all three at the return-of-capital margin at once.

The clean test is within jurisdiction over time. Cross-jurisdiction comparisons of ordering rules invite the objection that legal regimes proxy for correlated institutions, whereas a change in the distribution-tax rate, the 2003 cut foremost, moves γ while holding the rule fixed. The curvature, the payout lumpiness, and the earnings jump should all rescale with it. If they do not move together, the mechanism is rejected.

VIII. CONCLUSION

The paper's argument runs in one line. Retained earnings and contributed capital leave the firm on different tax terms; the difference sets a dollar-by-dollar hurdle for keeping projects, so equity composition governs what the firm divests and the earnings it expects; the ordering rule makes the tax unavoidable and the access choice indivisible, so optimal payout is binary; and capitalizing the resulting returns bends firm value at the regime threshold by exactly the distribution-tax rate. The homogeneity of book equity is therefore a substantive economic assumption, not a harmless convenience.

Three results follow, and they differ in kind. The first is behavioral and smooth: firms identical but for the mix hold different portfolios and earn different expected earnings, and holding those earnings fixed recovers the composition irrelevance of Guenther and Sansing (2006) as the special case that assumption creates. The ordering rule adds the other two. Payout becomes binary, so otherwise identical firms sort into different regimes by composition alone and distributions that reach paid-in capital arrive as discrete events rather than marginal adjustments. And value at the distribution margin is piecewise linear with a convex kink whose magnitude is the distribution-tax rate itself, independent of the firm's return parameters; under threshold uncertainty the corner becomes a smooth bend with the same integrated slope change. That magnitude is fixed by the tax system rather than fitted.

For residual income valuation the implications are constructive rather than destructive. Ohlson's framework survives the tax; what fails is the sufficiency of aggregate book value, since the normal-return benchmark applies different after-tax opportunity costs to the two components. Away from the distribution margin the relation stays linear once book value is split, so the framework needs one additional accounting state variable rather than a new architecture, and expected abnormal earnings become the product of a distribution policy that composition sets, giving economic content to part of what the linear dynamics would otherwise leave in the other-information variable. At the return-of-capital margin the linear form itself gives way, so a specification imposing a constant coefficient on composition pools two regimes whose slopes differ by γ and is misspecified precisely where composition matters most. The model locates that bend and fixes its magnitude, so tax-driven convexity can in principle be told apart from convexity arising from abandonment or growth options.

The scope is deliberately limited. The results are comparative statics comparing otherwise identical firms at a point in time, holding the post-distribution allocation fixed; access timing and the pace at which repeated repurchases bring the toll forward are questions for companion work. Within that scope the predictions are joint and falsifiable: payout, investment quality, and the valuation slope move together at the same boundary, scaled by the same rate.

One measurement point remains. The tax quantity driving the mechanism is earnings and profits, for which book retained earnings is the observable proxy (Section II; Appendix A), and the wedge between them is measurement error the tests inherit. The disclosure implication is not that the equity split be reported, since it already is, but that the tax attributes behind it, earnings and profits and the basis channel governing repurchase taxation, are what the mechanism would reward if observable. The thesis remains the institutional one the paper opened with: the distribution ordering rule, usually read as burying contributed capital, is what makes equity composition a priced feature of the firm.

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FIGURE 1

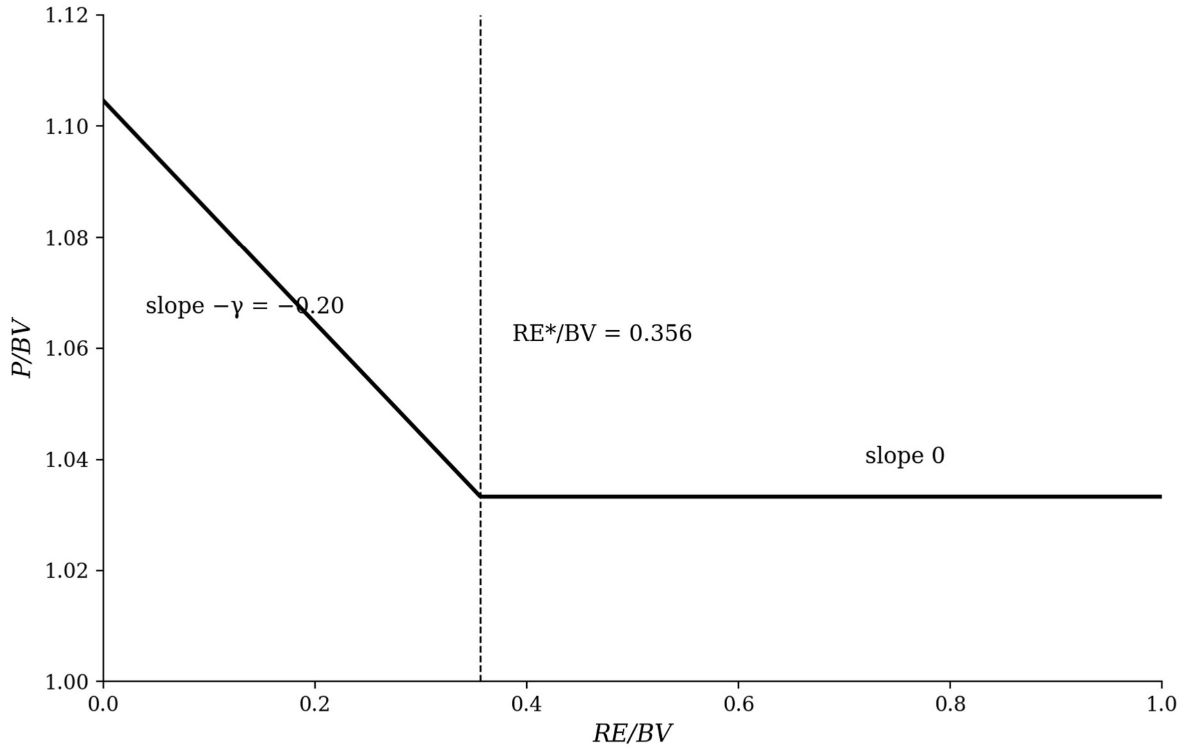

Figure 1. Firm value as a function of equity composition, at the parameterization of Table 2, column B ($\gamma = 0.20$). The horizontal axis is the retained-earnings share of book value, *RE/BV*; the vertical axis is firm value scaled by book value, *P/BV*. To the right of the regime threshold, at *RE/BV* = 0.356, the firm reallocates nothing and value is flat: in this retention regime, equity composition does not affect value. To the left, in the access regime, the firm returns capital to shareholders, and each additional dollar of retained earnings raises the tax cost of that reallocation, so value declines with slope $-\gamma = -0.20$. The two regimes meet in a convex kink at the threshold, where the slope changes by exactly γ, the distribution-tax rate; this change is independent of the return-distribution parameters and is the same in all three columns of Table 2. The relation is drawn at true scale.

TABLE 2

Table 2. Calibrated magnitudes at $\gamma = 0.20$ and a four percent risk-free rate. The regime threshold varies with the return-distribution parameters (a, δ); the kink magnitude does not, illustrating the invariance established by Propositions 5 and 6.

	(A) $a = 1$, $\delta = 0.04$	**(B) $a = 2$, $\delta = 0.02$**	**(C) $a = 3$, $\delta = 0.015$**
$\theta = \gamma(R_f - 1)/[\gamma(R_f - 1) + \delta]$	0.167	0.286	0.348
Baseline Distribution d_{RE}/BV (zero at this calibration: no project earns below the after-tax hurdle)	0	0	0
Access Distribution $d_C/BV = \theta^{1/a}$	0.167	0.535	0.703
Regime threshold $RE^*/BV = [a/(a + 1)]\theta^{1/a}$	0.083	0.356	0.527
Retention-regime slope	0	0	0
Access-regime slope	−0.20	−0.20	−0.20
Kink: γ	**0.20**	**0.20**	**0.20**

APPENDIX A

TAX PROVISIONS AND THE EFFECTIVE DISTRIBUTION TAX

Many tax systems distinguish distributions of earnings, taxed as dividends, from returns of contributed capital; this appendix focuses on the U.S. rules. Sections A through D state the relevant provisions, each closing with a note on its bearing on the ordering rule, and Section E draws out the implications for the effective rate γ: why the wedge between retained earnings and contributed capital survives the principal payout channels, and what observable features moderate it.

A. Retained Earnings versus Contributed Capital

The general rule is that equity distributions from accumulated or current earnings are taxable as dividends, whereas distributions from contributed capital are tax-free returns of capital; other systems, such as the United Kingdom, draw the same distinction. The United States formalizes it in 26 U.S. Code §316(a), under which a distribution is a dividend to the extent of current or accumulated earnings and profits regardless of the label the corporation applies; the ordering is mandatory by operation of law, not a firm election or a modeling convenience.

The Code does not precisely define earnings and profits; the concept measures a corporation's economic capacity to pay dividends and is economically related to accounting retained earnings, though the two measures differ in important cases: stock dividends reduce accounting retained earnings without reducing earnings and profits, and §312 governs property transactions separately. Accounting retained earnings therefore serves as an empirical proxy for tax earnings and profits, the variable through which the ordering rule formally operates. The proxy is reliable for firms whose retained earnings track accumulated distributable profits, but the divergence is largest for exactly the firms the model isolates, those with heavy contributed capital, buyback-driven or accumulated-deficit book equity, or histories of stock dividends and property distributions. For such firms accounting retained earnings can misstate the distributable base enough to place a firm on the wrong side of the threshold, not merely add noise. Empirical implementations of the Section VII predictions should use tax-based earnings and profits where it can be recovered, treating accounting retained earnings as an approximation whose error concentrates in the access regime.

26 U.S. Code §301(c) governs the taxation of equity distributions in three tiers: a distribution is a taxable dividend to the extent of current or accumulated earnings and profits; distributions in excess are tax-free returns of capital that reduce the shareholder's basis; and amounts beyond basis are taxed as gain. The dividend tier cannot be skipped. It applies before any return of capital, however the distribution is labeled, and returns of capital do not occur until earnings and profits are exhausted. For ordinary distributions, these rules support the ordering-rule mechanism directly.

B. Dividend Allowances

Many countries provide tax-favored dividend allowances, such as the U.K. zero-rate allowance, U.S. preferential rates on qualified dividends, and imputation credits. These lower the effective distribution tax rate γ but leave the ordering structure unchanged.

C. Share Repurchases (Treasury Stock)

Share repurchases are an alternative channel for returning equity to shareholders. Shareholders generally treat a repurchase, or stock redemption, as a sale of stock that produces a capital gain (26 U.S.*C.* §302(a)). In the United States, long-term capital gains are taxed at rates equal to or below the dividend rate and short-term gains at higher rates.

A repurchase also lets a shareholder offset the gain with tax basis, but this benefit is not unique to it: a shareholder who received dividends instead could apply the same basis against tax on a later sale. The basis offset is therefore not a complete escape unique to repurchasing; it changes the timing, channel, and taxable base of shareholder-level taxation, and repurchases, like dividends, generally produce a personal tax for taxable shareholders.

Share repurchases also reduce a company's taxable earnings and profits, but not dollar-for-dollar as a dividend does. When a repurchase qualifies as an exchange under 26 U.S.*C*. §302(a), §312(n)(7) limits the reduction in earnings and profits to the redeemed stock's ratable share of those earnings, capped at the redemption price; a repurchase that does not qualify as an exchange is taxed as a dividend. While retained earnings remain, either dividends or repurchases generally trigger a personal tax; once they are exhausted, further distributions can be returned tax-free as nondividend returns of capital.

D. Complete Liquidations

A complete liquidation generally treats payments as exchange proceeds for stock, taxing shareholder gain by reference to basis rather than applying the ordinary-distribution earnings-and-profits ordering rule (26 U.S.*C*. §331), removing the direct relevance of the distinction; some indirect relevance survives because shareholders' basis is often correlated with contributed capital. Complete liquidations therefore fall outside the central ordinary-distribution mechanism the model analyzes.

E. Implications for the Effective Rate γ

This subsection grounds the repurchase discussion of Section V.C in the governing provisions, states its identity formally, and develops γ as a realization-weighted effective rate. The argument itself is made in the body and is not repeated. The setting is the standing objection that share repurchases let firms return cash without paying the distribution tax: under the view that the tax on retained earnings is capitalized because it is borne whenever those earnings are eventually distributed (King, 1977; Auerbach, 1979), repurchases taxed as sales rather than dividends appear to provide an exit. That reply is correct about the dividend tax specifically and leaves untouched the tax structure the model relies on.

A repurchase does not let shareholders extract value free of tax; it converts the dividend tax into a capital gains tax on the value accumulated above contributed capital. The amount a selling shareholder realizes above basis is taxed, and basis, for the equity originally paid in, is contributed capital.[25] After shares trade, individual basis diverges from contributed capital, but the divergence never enlarges the untaxed total, because every dollar of basis above contributed capital was created by a gain on which a predecessor was taxed. Summed over a complete history, the taxed amounts, dividends in full and gains net of basis, collapse to the value distributed less the capital paid in: retained earnings bear the shareholder-level tax exactly once on their way out, and contributed capital is the only amount that never bears it, recovered as basis in a repurchase and returned as a nondividend distribution once earnings and profits are exhausted under the ordering rule. Two institutional facts carry the channel-mix half of the claim: a dividend is taxed in full even for a holder who has not yet recovered his basis, his relief arriving only at sale, and gains and losses enter the sum symmetrically, so actual limits on loss deductibility only push the taxed total higher.

The two channels do not tax an identical base, since a repurchase reaches the abnormal-earnings premium capitalized into price and reaches it at once, where a dividend reaches retained earnings as they are distributed; but under either channel the wedge lies between contributed

[25] The aggregate correspondence is an accounting identity. Let a share be issued for contributed capital c and pass through holders 1 through n at prices p1 through pn, with the firm repurchasing at pn and with p0 = c. Let holder i also receive any dividends the firm pays while he holds the share. Holder i is taxed on his dividends plus the gain pi − pi−1, with gains and losses entering symmetrically, so the taxed amounts sum to the dividends paid plus pn − c. The number of intermediate trades, the path of individual bases, and the mix of dividends and repurchases are therefore irrelevant to the cumulative taxed amount, which equals the value above contributed capital however that value arose. The complement is the amount that never bears tax, and it is exactly c, whatever the bases of the holders alive at any date. Where shares are issued at more than one date, c is cumulative net contributed capital.

capital and the value above it, not between two payout instruments. The model implements that wedge as the tax γ on book retained earnings, the earnings-and-profits proxy of subsection A. The claim here is the conceptual one, that γ is the right effective rate and contributed capital the right floor whether value returns as a dividend or as a gain, not that the formal base moves onto the market premium.

The identity fixes the total; two observations govern its incidence within a period, who pays and when, without altering the sum. Aggregate basis at any date equals contributed capital plus the gains already realized and taxed along the chain, so the shield above the floor is finite and cannot be enlarged except by paying for it. And a repurchase retires the basis it recovers while a firm cannot choose the basis of its sellers, so to the extent that higher-basis holders realize first, deeper repurchases meet lower-basis holders and the marginal repurchased dollar is taxed closer to the dividend rate.[26] A legal constraint runs alongside the economic one, since a repurchase that fails exchange treatment under §302(a) is taxed as a dividend outright, as subsection C notes. The cost of the channel therefore rises precisely at the depth where the ordering rule binds. Turnover does not relax it either, since basis above contributed capital is minted only when a holder realizes a gain and pays the tax, so waiting for the shield to rebuild collects the same toll from the sellers who rebuild it. The channel relabels the tax and retimes it; no policy of depth or patience shrinks the total.

Two qualifications bound this claim, and together they locate where it has force. The capital gains channel is leakier than the dividend channel: long-term gains are often taxed at a lower rate, gains may be deferred, and a step-up in basis at death extinguishes the tax on

[26] The monotonicity is immediate. Order shares by holder basis b. A holder's tax on sale is decreasing in b, so at any offer price the highest-basis shares are tendered first, and a repurchase of size R draws the R highest-basis shares. The tax per repurchased dollar, being the price less the basis of the marginal tendering holder, is therefore nondecreasing in R. The argument requires only that tendering be ordered by basis, not any particular distribution of basis across holders.

appreciation held until then. For a clientele that holds until death, value above contributed capital is never realized and is wiped at transfer, so the wedge does not merely compress for such holders, it closes. The effective γ is thus a blended, realization-weighted rate that falls as investors' horizons lengthen or holdings shift toward non-taxable hands.

Three objects should be kept distinct: the statutory rate on qualified dividends or long-term gains; the model's effective rate γ, the blended cost of crossing the retained-earnings layer; and any empirical proxy built from ownership, payout channel, and horizon. For illustration, γ is approximately the dividend weight times the dividend rate, plus the repurchase weight times the taxable-gain share of repurchase proceeds times the gains rate, with clienteles and horizons setting the weights. The model's predictions scale with this rate and vanish only at $\gamma = 0$, so the composition effect has force where the price-setting clientele contains taxable holders who realize value at the return-of-capital margin, the same population the treatment of investor heterogeneity points to in Sections III.B and VII.

Repurchases have not displaced the dividend channel the ordering rule governs. U.S. dividend payments remain large and have continued to grow, so the mechanism describes a primary mode of returning earnings rather than a relic of the period before repurchases became common (Floyd, Li, & Skinner, 2015). Because qualified dividends and long-term capital gains now face similar shareholder rates, the choice between the two channels turns less on the rate than on the contributed-capital floor both channels share.

The same logic supplies a second moderator, on the investor side. Because the slope below the threshold is the effective γ of the firm's investor clientele, the kink should be sharper for firms whose investor base is more heavily taxed and should flatten as ownership shifts toward tax-exempt and foreign hands. The 2003 reduction in shareholder taxes lowered the effective tax on returning value above contributed capital for taxable investors while leaving tax-exempt holders unaffected: the kink should attenuate after 2003 for firms held by taxable investors and not for firms held by exempt ones, a comparative static that ties the kink's tax-rate dependence to the change in law rather than to cross-sectional variation in ownership, and so does not rest on whether clienteles jump at the access margin.

The clientele's blended rate also fixes where the kink can appear: it requires taxed investors with weight in the price who realize value at the return-of-capital margin, since for a firm priced entirely by untaxed holders the slope below the threshold is zero and no kink arises. This is a conditional boundary, not a claim that the firms of interest satisfy it.[27]

A natural worry is that clientele sorting selects low-tax investors into the very firms transacting at the return-of-capital margin, shrinking the effective rate where the kink should appear. The ordering rule blocks that selection at this margin: a distribution reaching contributed capital is taxed as a dividend to the extent of earnings and profits, at the dividend rate, not as a deferred capital gain, so the low-rate clientele is the one sorting into firms that avoid the margin through repurchases rather than the one pricing firms at it. The remaining cross-firm variation in the clientele's blended rate is itself an observable moderator of the kink's sharpness.

APPENDIX B

This appendix collects the formal machinery and the proofs. All functions, definitions, and parameter restrictions are as stated in the setup of Section IV.A; the appendix works in its own notation, recorded below.

[27] The clientele evidence sorts on dividend yield (Elton & Gruber, 1970); return-of-capital and nondividend distributions are not that population, so taxable investors pricing firms at the return-of-capital margin are consistent with the clientele literature rather than in tension with it. The kink's identification does not rely on this, however; it requires only that taxed investors carry weight in the firm's price.

Notation used in Appendix B.

BV, *RE*, *C*: book value, retained earnings, and contributed capital ($BV = RE + C$).

x: position in the ranked return schedule, equivalently the cumulative capital distributed and reinvested externally at the risk-free rate.

$f(x)$, $g(x)$: marginal after-tax return on retained capital and on distributed capital, respectively.

d: total distribution. d_R: Baseline Distribution. d_L: Access Distribution.

RE^*: regime threshold. $\lambda = (R_f - 1)\gamma \,/\, [(R_f - 1)\gamma + \delta]$.

$TR^*(RE)$: optimized after-tax return flow. $V(RE)$: firm value.

Correspondence with the main text: it writes d_{RE} *for dR,* d_C *for dL, and* θ *for* λ.

Part A. The Baseline Distribution

Lemma 1: Let all functions and definitions be as stated in Section IV.A of the text. Then we have that $\exists!\, d_R \in [0, BV]$ s.t. $f(d_R) = \left(R_f - 1\right)(1-\gamma)$.

Proof: Note that $f\colon [0, BV] \to \mathbb{R}$ given by $f(x) = \left(\mathrm{R}_f - 1\right)(1-\gamma) - \sigma + \left(\gamma\left(\mathrm{R}_f - 1\right) + \delta + \sigma\right)\left(\frac{x}{BV}\right)^{\mathrm{a}}$ with $\mathrm{a} > 0$ is a sum of continuous functions on the domain $[0, BV]$, and hence f is continuous on $[0, BV]$. Additionally, since f is strictly increasing in x, we have that f is injective. First, suppose $\sigma = 0$. Then, choose $d_\mathrm{R} = 0$. Then, $d_\mathrm{R} \in [0, BV]$ s.t. $f(d_\mathrm{R}) = f(0) = \left(\mathrm{R}_f - 1\right)(1-\gamma)$. Since f is injective, it follows that if for some $d_\mathrm{R}' \in [0, BV]$ we have that $f(d_\mathrm{R}') = f(d_\mathrm{R})$, then $d_\mathrm{R}' = d_\mathrm{R}$. Hence, we have that d_R is unique, as desired. Alternatively, assume $\sigma > 0$. Since $f(0) = \left(\mathrm{R}_f - 1\right)(1-\gamma) - \sigma$ and $f(BV) = \left(\mathrm{R}_f - 1\right) + \delta$ where $\gamma, \sigma > 0$ and $\delta \geq 0$, we have that $f(0) = \left(\mathrm{R}_f - 1\right)(1-\gamma) - \sigma < \left(\mathrm{R}_f - 1\right)(1-\gamma) < \left(\mathrm{R}_f - 1\right) \leq f(BV) = \left(\mathrm{R}_f - 1\right) + \delta$. Then, by the Intermediate Value Theorem we have that $\exists d_\mathrm{R} \in (0, BV)$ s.t. $f(d_\mathrm{R}) = \left(\mathrm{R}_f - 1\right)(1-\gamma)$, and hence $d_\mathrm{R} \in [0, BV]$. Additionally, it follows from the injectivity of f that if for some $d_{\mathrm{R}}' \in [0, BV]$ we have that $f(d_{\mathrm{R}}') = f(d_{\mathrm{R}})$, then $d_{\mathrm{R}}' = d_{\mathrm{R}}$. Hence, d_{R} is unique, as desired.

Proof of Proposition 1 (Baseline Distribution)

Proof: First, consider the case where $\sigma = 0$. Then $f(0) = (\mathrm{R}_f - 1)(1 - \gamma)$, and since by Lemma 1 we have that d_{R} exists and is unique, we have that $d_{\mathrm{R}} = 0$. Then let D be the set of all $d \geq 0$ s.t. $d < d_{\mathrm{R}}$. Note that since d_{R} is unique, it follows that $d_{\mathrm{R}} = 0 \Rightarrow \mathrm{D} = \emptyset$. Hence, it follows, vacuously, that $\forall d \in \mathrm{D}$ we have that $TR(d) < TR(d_{\mathrm{R}})$, as desired. Next, consider the case where $\sigma > 0$. Then, by Lemma 1 and the Intermediate Value Theorem, we have that $0 < d_{\mathrm{R}} < BV$. Then let $0 \leq d_0 < d_{\mathrm{R}}$. Note that since f is strictly increasing in x and $f(d_{\mathrm{R}}) = (\mathrm{R}_f - 1)(1 - \gamma)$, we have that $\forall x$ satisfying $d_0 < x < d_{\mathrm{R}}$, $f(x) < (\mathrm{R}_f - 1)(1 - \gamma)$. Since $g(x) \geq (\mathrm{R}_f - 1)(1 - \gamma)\ \forall x$, we have that $\forall x$ satisfying $d_0 < x < d_{\mathrm{R}}$, $f(x) < g(x)$. Then we have that $TR(d_{\mathrm{R}}) = \int_{d_{\mathrm{R}}}^{BV} f(x)\mathrm{d}x + \int_0^{d_{\mathrm{R}}} g(x)\mathrm{d}x = \int_{d_{\mathrm{R}}}^{BV} f(x)\mathrm{d}x + \int_{d_0}^{d_{\mathrm{R}}} g(x)\mathrm{d}x +$
$\int_0^{d_0} g(x)\mathrm{d}x > \int_{d_{\mathrm{R}}}^{BV} f(x)\mathrm{d}x + \int_{d_0}^{d_{\mathrm{R}}} f(x)\mathrm{d}x + \int_0^{d_0} g(x)\mathrm{d}x = \int_{d_0}^{BV} f(x)\mathrm{d}x + \int_0^{d_0} g(x)\mathrm{d}x =$
$TR(d_0)$. Hence, $TR(d_0) < TR(d_{\mathrm{R}})$, as desired.

Part B. The Access Distribution

Lemma 2: Let all functions and definitions be as stated in Section IV.A of the text, and let $\sigma = 0$. Then we have that $\exists!\, d_L \in (0, BV]$ s.t. $f(d_L) = R_f - 1$.

Proof: Note that $f\colon [0, BV] \to \mathbb{R}$ given by $f(x) = (\mathrm{R}_f - 1)(1 - \gamma) + \left(\gamma(\mathrm{R}_f - 1) + \delta\right)\left(\frac{x}{BV}\right)^{\mathrm{a}}$ with $\mathrm{a} > 0$ is a sum of continuous functions on the domain $[0, BV]$, and hence f is continuous on $[0, BV]$. Additionally, since f is strictly increasing in x, we have that f is injective. First, consider the case where $\delta = 0$. Then choose $d_{\mathrm{L}} = BV$. Then $d_{\mathrm{L}} \in (0, BV]$ and

$f(d_{\mathrm{L}}) = f(BV) = \mathrm{R}_f - 1$. Additionally, since f is injective, it follows that if for some $d_{\mathrm{L}}' \in (0, BV]$ we have that $f(d_{\mathrm{L}}') = f(d_{\mathrm{L}})$, then $d_{\mathrm{L}}' = d_{\mathrm{L}}$. Hence, d_{L} is unique, as desired. Next, assume $\delta > 0$. Since $f(0) = (\mathrm{R}_f - 1)(1 - \gamma)$ and $f(BV) = (\mathrm{R}_f - 1) + \delta$ where $\gamma > 0$ and $\delta > 0$, we have that $f(0) = (\mathrm{R}_f - 1)(1 - \gamma) < (\mathrm{R}_f - 1) < f(BV) = (\mathrm{R}_f - 1) + \delta$. Then, by the Intermediate Value Theorem we have that $\exists d_{\mathrm{L}} \in (0, BV)$ s.t. $f(d_{\mathrm{L}}) = \mathrm{R}_f - 1$, and thus it follows that $\exists d_{\mathrm{L}} \in (0, BV]$ s.t. $f(d_{\mathrm{L}}) = \mathrm{R}_f - 1$. Additionally, since f is injective, it follows that if for some $d_{\mathrm{L}}' \in (0, BV]$ we have that $f(d_{\mathrm{L}}') = f(d_{\mathrm{L}})$, then $d_{\mathrm{L}}' = d_{\mathrm{L}}$. Hence, d_{L} is unique, as desired.

Proof of Proposition 2 (Discreteness)

The proof proceeds in three steps. First, continuity of total returns over the closed interval ensures that an optimum exists. Second, no distribution strictly between zero and the Access Distribution is optimal, because such a distribution pays the tax toll without reaching the tax-free contributed-capital layer. Third, no distribution beyond the Access Distribution is optimal, because it sheds projects earning more than the tax-free external return. The two exclusions leave only the endpoints.

Proof: We begin by showing that $TR(d)$ is continuous over its domain, $[0, BV]$, and that D^* is non-empty.

Recall that $TR(d) = \int_d^{BV} f(x)\mathrm{dx} + \int_0^d g(x)\mathrm{dx}$. Note that $\int_0^d g(x)\mathrm{dx} = \int_0^{RE} g(x)\mathrm{dx} + \int_{RE}^d g(x)\mathrm{dx}$ if $d \geq RE$. Then for the interval $[RE, BV]$, $TR(d)$ is a sum of integrals of continuous functions, and so we have by the Fundamental Theorem of Calculus that $TR(d)$ is continuous over $[RE, BV]$. If $d \leq RE$, then $\int_0^d g(x)\mathrm{dx}$ is an integral of a continuous function, and hence for the interval $[0, RE]$, $TR(d)$ is the sum of integrals of continuous functions and hence by the

Fundamental Theorem of Calculus $TR(d)$ is continuous over $[0, RE]$. As a final check, consider the point where $d = RE$. Since only $g(x)$is discontinuous at that point, it suffices to show that $\mathrm{G}(d) = \int_0^d g(x)\mathrm{dx}$ is continuous at the point $d = RE$. Note that $\lim_{d \to RE^-} \int_0^d g(x)\mathrm{dx} = \lim_{d \to RE^-} \int_0^d (\mathrm{R}_f - 1)(1-\gamma)\mathrm{dx} = \lim_{d \to RE^-} x(\mathrm{R}_f - 1)(1-\gamma)\Big|_0^d = \lim_{d \to RE^-} d(\mathrm{R}_f - 1)(1-\gamma) - 0 = \lim_{d \to RE^-} d(\mathrm{R}_f - 1)(1-\gamma) = RE(\mathrm{R}_f - 1)(1-\gamma)$. Meanwhile, we have that $\lim_{d \to RE^+} \int_0^d g(x)\mathrm{dx} = \lim_{d \to RE^+} \int_0^{RE} (\mathrm{R}_f - 1)(1-\gamma)\mathrm{dx} + \int_{RE}^d (\mathrm{R}_f - 1)\mathrm{dx} = \lim_{d \to RE^+} x(\mathrm{R}_f - 1)(1-\gamma)\Big|_0^{RE} + x(\mathrm{R}_f - 1)\Big|_{RE}^d = \lim_{d \to RE^+} RE(\mathrm{R}_f - 1)(1-\gamma) - 0 + d(\mathrm{R}_f - 1) - RE(\mathrm{R}_f - 1) = \lim_{d \to RE^+} RE(\mathrm{R}_f - 1)(1-\gamma) + d(\mathrm{R}_f - 1) - RE(\mathrm{R}_f - 1) = RE(\mathrm{R}_f - 1)(1-\gamma) + RE(\mathrm{R}_f - 1) - RE(\mathrm{R}_f - 1) = RE(\mathrm{R}_f - 1)(1-\gamma) + 0 = RE(\mathrm{R}_f - 1)(1-\gamma) = \lim_{d \to RE^-} \int_0^d g(x)\mathrm{dx}$. Since $\lim_{d \to RE^-} \int_0^d g(x)\mathrm{dx} = \lim_{d \to RE^+} \int_0^d g(x)\mathrm{dx} = RE(\mathrm{R}_f - 1)(1-\gamma)$, it follows that $\lim_{d \to RE} \int_0^d g(x)\mathrm{dx} = RE(\mathrm{R}_f - 1)(1-\gamma) = \lim_{d \to RE} \mathrm{G}(d)$. Further note that for the actual value at the point, we have $\mathrm{G}(RE) = x(\mathrm{R}_f - 1)(1-\gamma)\Big|_0^{RE} = RE(\mathrm{R}_f - 1)(1-\gamma) = \lim_{d \to RE} \mathrm{G}(d)$. Since the value of the function at the point $d = RE$ equals the limit of the functions value approaching that point, it follows that $\mathrm{G}(d)$ is continuous at $d = RE$. Since $\mathrm{F}(d) = \int_d^{BV} f(x)\mathrm{dx}$ is continuous at $d = RE$ by the Fundamental Theorem of Calculus, it follows that $TR(d) = \mathrm{F}(d) + \mathrm{G}(d)$ is the sum of two functions that are continuous at $d = RE$, and hence $TR(d)$ is continuous at $d = RE$. Then $TR(d)$ is continuous over the closed interval $[0, BV]$.

Then, by the Extreme Value Theorem, we have that $\exists d^* \in [0, BV]$ s.t. $\forall d \in [0, BV], TR(d^*) \geq TR(d)$. Then $d^* \in \underset{d \in [0,BV]}{\operatorname{argmax}} TR(d) = \mathrm{D}^*$, and so D^* is non-empty, as desired.

Next, since D^* is non-empty, let $d^* \in D^*$. We first show that $d^* \notin (0, d_L)$. Suppose, by way of contradiction, that $0 < d^* < d_L$. Recall that $f: [0, BV] \to \mathbb{R}$ is given by $f(x) = (R_f - 1)(1 - \gamma) + (\gamma(R_f - 1) + \delta)\left(\frac{x}{BV}\right)^a$, with $a, \gamma > 0$ and $\delta \geq 0$, and so f is strictly increasing in x. Further, recall that by definition of the Access Distribution, $f(d_L) = R_f - 1$. Then, we have that $d^* < d_L \Rightarrow f(d^*) < f(d_L) = (R_f - 1)$. Suppose that $d^* \leq RE$, then $\forall d \leq d^*$ we have that $g(d) = (R_f - 1)(1 - \gamma) < (R_f - 1)(1 - \gamma) + (\gamma(R_f - 1) + \delta)\left(\frac{x}{BV}\right)^a = f(d)$. Then $\int_0^{d^*} g(x)dx < \int_0^{d^*} f(x)dx$. Thus, $TR(d^*) = \int_0^{d^*} g(x)dx + \int_{d^*}^{BV} f(x)dx < \int_0^{d^*} f(x)dx + \int_{d^*}^{BV} f(x)dx = TR(0)$. Then $TR(0) > TR(d^*)$, contradicting the optimality of d^*. Hence, $0 < d^* < d_L$, with $d^* \leq RE$ is not possible. Next, consider the case where $RE < d^* < d_L$. Note that $\forall d\ s.t.\ RE < d < d^*$, we have that $g(d) = R_f - 1$. More particularly, note that $\forall d\ s.t.\ d^* < d < d_L$, we have that $g(d) = R_f - 1$. Additionally, since f is strictly increasing and $f(d^*) < f(d_L) = R_f - 1$, we have that $\forall d\ s.t. d^* < d < d_L, f(d) < g(d) = R_f - 1$. Then, $\int_{d^*}^{d_L} f(x)dx < \int_{d^*}^{d_L} g(x)dx$. We then have that $TR(d^*) = \int_0^{d^*} g(x)dx + \int_{d^*}^{BV} f(x)dx = \int_0^{d^*} g(x)dx + \int_{d^*}^{d_L} f(x)dx + \int_{d_L}^{BV} f(x)dx < \int_0^{d^*} g(x)dx + \int_{d^*}^{d_L} g(x)dx + \int_{d_L}^{BV} f(x)dx = \int_0^{d_L} g(x)dx + \int_{d_L}^{BV} f(x)dx = TR(d_L)$. Then $TR(d^*) < TR(d_L)$, contradicting the optimality of d^*. Hence, $0 < d^* < d_L$, with $d^* > RE$ is not possible. Thus we have that $d^* \notin (0, d_L)$, as desired.

Next, we show that $d^* \notin (d_L, BV]$. Suppose, by way of contradiction, that $d_L < d^* \leq BV$. First, consider the case where $d^* > RE$. Note that since f is strictly increasing, we have that

$\forall d\ s.t.\ d_L < d < d^*, f(d) > f(d_L) = R_f - 1 \geq g(d)$. Then $\int_{d_L}^{d^*} f(x)dx > \int_{d_L}^{d^*} g(x)dx$. Then we have that $TR(d^*) = \int_0^{d^*} g(x)dx + \int_{d^*}^{BV} f(x)dx = \int_0^{d_L} g(x)dx + \int_{d_L}^{d^*} g(x)dx + \int_{d^*}^{BV} f(x)dx < \int_0^{d_L} g(x)dx + \int_{d_L}^{d^*} f(x)dx + \int_{d^*}^{BV} f(x)dx = \int_0^{d_L} g(x)dx + \int_{d_L}^{BV} f(x)dx = TR(d_L)$. Then $TR(d^*) < TR(d_L)$, contradicting the optimality of d^*. Hence, $d_L < d^* \leq BV$, with $d^* > RE$ is not possible. Then consider the case where $d^* \leq RE$. Then, as before, $\forall d \leq d^*$ we have that $g(d) = (R_f - 1)(1-\gamma) < (R_f - 1)(1-\gamma) + (\gamma(R_f - 1) + \delta)\left(\frac{x}{BV}\right)^a = f(d)$. Thus $\int_0^{d^*} g(x)dx < \int_0^{d^*} f(x)dx$. Thus, $TR(d^*) = \int_0^{d^*} g(x)dx + \int_{d^*}^{BV} f(x)dx < \int_0^{d^*} f(x)dx + \int_{d^*}^{BV} f(x)dx = TR(0)$. Then $TR(0) > TR(d^*)$, contradicting the optimality of d^*. $d_L < d^* \leq BV$, with $d^* > RE$ is not possible. Then we have that $d^* \notin (d_L, BV]$, as desired.

Since d^* exists, with $d^* \in [0, BV]$, but $d^* \notin (0, d_L)$ and $d^* \notin (d_L, BV]$, we have that $d^* \in \{0, d_L\}$. Since d^* was an arbitrary element of D^*, it follows that $D^* \subseteq \{0, d_L\}$, as desired.

Proof of Proposition 3 (Optimal Distribution)

Proof: Recall that by Proposition 2, we have that $\mathrm{D}^* = \underset{d \in [0,BV]}{\operatorname{argmax}} TR(d)$ is non-empty with $\mathrm{D}^* \subseteq \{0, d_\mathrm{L}\}$. Further, by Lemma 2, d_L exists and by definition satisfies $f(d_\mathrm{L}) = \mathrm{R}_f - 1$. Note that $f(d_\mathrm{L}) = \mathrm{R}_f - 1 \Rightarrow \mathrm{R}_f - 1 = (\mathrm{R}_f - 1)(1-\gamma) + (\gamma(\mathrm{R}_f - 1) + \delta)\left(\frac{d_\mathrm{L}}{BV}\right)^\mathrm{a}$, so $\gamma(\mathrm{R}_f - 1) = (\gamma(\mathrm{R}_f - 1) + \delta)\left(\frac{d_\mathrm{L}}{BV}\right)^\mathrm{a}$. Then $\frac{\gamma(\mathrm{R}_f - 1)}{\gamma(\mathrm{R}_f - 1) + \delta} = \left(\frac{d_\mathrm{L}}{BV}\right)^\mathrm{a}$, so $\left(\frac{\gamma(\mathrm{R}_f - 1)}{\gamma(\mathrm{R}_f - 1) + \delta}\right)^{\frac{1}{\mathrm{a}}} = \frac{d_\mathrm{L}}{BV}$ and $d_\mathrm{L} = BV\left(\frac{\gamma(\mathrm{R}_f - 1)}{\gamma(\mathrm{R}_f - 1) + \delta}\right)^{\frac{1}{\mathrm{a}}}$. Since $\mathrm{a} > 0$, d_L is well-defined for all parameter values. Additionally, using the definition of λ, we can rewrite $d_\mathrm{L} = BV\lambda^{\frac{1}{\mathrm{a}}}$, as desired.

We first consider the case where $0 < d_{\mathrm{L}} \leq RE$. Since $d_{\mathrm{L}} = BV\lambda^{\frac{1}{\mathrm{a}}}$, it follows that $d_{\mathrm{L}} \leq RE \Leftrightarrow$ $BV\lambda^{\frac{1}{\mathrm{a}}} \leq RE \Leftrightarrow \frac{RE}{BV} \geq \lambda^{\frac{1}{\mathrm{a}}}$. Note that since $d_{\mathrm{L}} \leq RE$, $\forall d \leq d_{\mathrm{L}}$ we have that $g(d) = (\mathrm{R}_f - 1)(1-\gamma) < (\mathrm{R}_f - 1)(1-\gamma) + (\gamma(\mathrm{R}_f - 1) + \delta)\left(\frac{x}{BV}\right)^{\mathrm{a}} = f(d)$. Then $\int_0^{d_{\mathrm{L}}} g(x)\mathrm{dx} <$ $\int_0^{d_{\mathrm{L}}} f(x)\mathrm{dx}$. Thus, $TR(d_{\mathrm{L}}) = \int_0^{d_{\mathrm{L}}} g(x)\mathrm{dx} + \int_{d_{\mathrm{L}}}^{BV} f(x)\mathrm{dx} < \int_0^{d_{\mathrm{L}}} f(x)\mathrm{dx} + \int_{d_L}^{BV} f(x)\mathrm{dx} = TR(0)$. Thus $TR(0) > TR(d_{\mathrm{L}})$ and so $d_{\mathrm{L}} \notin \mathrm{D}^*$. Since, by Proposition 2, we have that $\mathrm{D}^* \subseteq \{0, d_{\mathrm{L}}\}$ and that D^* is non-empty, we then have that $d_{\mathrm{L}} \notin \mathrm{D}^* \Rightarrow 0 \in \mathrm{D}^*$. Then $d_{\mathrm{L}} \leq RE \Rightarrow 0 \in \mathrm{D}^*$, with $d_{\mathrm{L}} \leq$ $RE \Leftrightarrow \frac{RE}{BV} \geq \lambda^{\frac{1}{\mathrm{a}}}$, such that $\frac{RE}{BV} \geq \lambda^{\frac{1}{\mathrm{a}}} \Rightarrow 0 \in \mathrm{D}^*$. Note that since $\mathrm{a} > 0$, we have that $\frac{\mathrm{a}}{\mathrm{a}+1} < 1$, and so $\frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} < \lambda^{\frac{1}{\mathrm{a}}}$, and so $\frac{RE}{BV} \geq \lambda^{\frac{1}{\mathrm{a}}} \Rightarrow \frac{RE}{BV} > \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$, such that $d_{\mathrm{L}} \leq RE \Rightarrow \frac{RE}{BV} > \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$. Then we have that under the condition $d_{\mathrm{L}} \leq RE$, the statement $\frac{RE}{BV} \leq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \Rightarrow d_{\mathrm{L}} \in \mathrm{D}^*$ holds vacuously, as $\frac{RE}{BV} \leq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$ contradicts $d_{\mathrm{L}} \leq RE$. The statement $d_{\mathrm{L}} \in \mathrm{D}^* \Rightarrow \frac{RE}{BV} \leq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$ holds vacuously as well, as $d_{\mathrm{L}} \in \mathrm{D}^*$ also contradicts $d_{\mathrm{L}} \leq RE$. Thus, $d_{\mathrm{L}} \in \mathrm{D}^* \Leftrightarrow \frac{RE}{BV} \leq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$ in this case as desired. Next, notice that since $\frac{RE}{BV} > \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \Rightarrow \frac{RE}{BV} \geq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$, it follows that if $d_{\mathrm{L}} \leq RE$, we have that both $0 \in \mathrm{D}^*$ and $\frac{RE}{BV} \geq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$ are always true, and hence the statement $0 \in \mathrm{D}^* \Leftrightarrow \frac{RE}{BV} \geq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$ holds. Additionally, note that since $\frac{RE}{BV} = \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$ contradicts $d_{\mathrm{L}} \leq RE$, the statement $\frac{RE}{BV} = \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \Rightarrow \mathrm{D}^* = \{0, d_{\mathrm{L}}\}$ holds vacuously. Additionally, since $d_{\mathrm{L}} \in \mathrm{D}^*$ also contradicts $d_{\mathrm{L}} \leq RE$, the statement $\mathrm{D}^* = \{0, d_{\mathrm{L}}\} \Rightarrow \frac{RE}{BV} = \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$ holds vacuously as well. Then the statement $\mathrm{D}^* = \{0, d_{\mathrm{L}}\} \Leftrightarrow \frac{RE}{BV} = \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$ holds. Thus we have that in the case where $d_{\mathrm{L}} \leq RE$, the proposition holds, as desired.

Next consider the case where $d_{\mathrm{L}} > RE$. Then we have that $d_{\mathrm{L}} \in (RE, BV]$. Note that since $TR(d_{\mathrm{L}}) = \int_0^{d_{\mathrm{L}}} g(x)\mathrm{dx} + \int_{d_{\mathrm{L}}}^{BV} f(x)\mathrm{dx}$ while $TR(0) = \int_0^{BV} f(x)\mathrm{dx} = \int_0^{d_{\mathrm{L}}} f(x)\mathrm{dx} + \int_{d_{\mathrm{L}}}^{BV} f(x)\mathrm{dx}$, and so $TR(d_{\mathrm{L}}) \geq TR(0)$ if and only if $\int_0^{d_{\mathrm{L}}} g(x)\mathrm{dx} \geq \int_0^{d_{\mathrm{L}}} f(x)\mathrm{dx}$. Note that since $d_{\mathrm{L}} > RE$, we have that $\int_0^{d_{\mathrm{L}}} g(x)\mathrm{dx} = \int_0^{RE} g(x)\mathrm{dx} + \int_{RE}^{d_{\mathrm{L}}} g(x)\mathrm{dx} = \int_0^{RE} (\mathrm{R}_f - 1)(1-\gamma)\mathrm{dx} + \int_{RE}^{d_{\mathrm{L}}} \mathrm{R}_f - 1\mathrm{dx} = (\mathrm{R}_f - 1)(1-\gamma)x \Big|_0^{RE} + (\mathrm{R}_f - 1)x \Big|_{RE}^{d_{\mathrm{L}}} = (\mathrm{R}_f - 1)(1-\gamma)RE - (\mathrm{R}_f - 1)(1-\gamma)(0) + (\mathrm{R}_f - 1)d_{\mathrm{L}} - (\mathrm{R}_f - 1)RE = (\mathrm{R}_f - 1)d_{\mathrm{L}} - \gamma(\mathrm{R}_f - 1)RE = (\mathrm{R}_f - 1)BV\lambda^{\frac{1}{\mathrm{a}}} - \gamma(\mathrm{R}_f - 1)RE$. Meanwhile, we have that $\int_0^{d_{\mathrm{L}}} f(x)\mathrm{dx} = \int_0^{d_{\mathrm{L}}} (\mathrm{R}_f - 1)(1-\gamma) + (\gamma(\mathrm{R}_f - 1) + \delta)\left(\frac{x}{BV}\right)^{\mathrm{a}} \mathrm{dx} = (\mathrm{R}_f - 1)(1-\gamma)x + \frac{BV(\gamma(\mathrm{R}_f-1)+\delta)\left(\frac{x}{BV}\right)^{\mathrm{a}+1}}{\mathrm{a}+1}\Bigg|_0^{d_{\mathrm{L}}} = (\mathrm{R}_f - 1)(1-\gamma)d_{\mathrm{L}} + \frac{BV(\gamma(\mathrm{R}_f-1)+\delta)\left(\frac{d_{\mathrm{L}}}{BV}\right)^{\mathrm{a}+1}}{\mathrm{a}+1} - (\mathrm{R}_f - 1)(1-\gamma)(0) - \frac{BV(\gamma(\mathrm{R}_f-1)+\delta)\left(\frac{0}{BV}\right)^{\mathrm{a}+1}}{\mathrm{a}+1} = (\mathrm{R}_f - 1)(1-\gamma)BV\lambda^{\frac{1}{\mathrm{a}}} + \frac{BV(\gamma(\mathrm{R}_f-1)+\delta)\left(\frac{BV\lambda^{\frac{1}{\mathrm{a}}}}{BV}\right)^{\mathrm{a}+1}}{\mathrm{a}+1} = (\mathrm{R}_f - 1)(1-\gamma)BV\lambda^{\frac{1}{\mathrm{a}}} + \frac{BV(\gamma(\mathrm{R}_f-1)+\delta)\lambda^{\frac{\mathrm{a}+1}{\mathrm{a}}}}{\mathrm{a}+1}$. Then $\int_0^{d_{\mathrm{L}}} g(x)\mathrm{dx} \geq \int_0^{d_{\mathrm{L}}} f(x)\mathrm{dx}$ if and only if $(\mathrm{R}_f - 1)BV\lambda^{\frac{1}{\mathrm{a}}} - \gamma(\mathrm{R}_f - 1)RE \geq (\mathrm{R}_f - 1)(1-\gamma)BV\lambda^{\frac{1}{\mathrm{a}}} + \frac{BV(\gamma(\mathrm{R}_f-1)+\delta)\lambda^{\frac{\mathrm{a}+1}{\mathrm{a}}}}{\mathrm{a}+1}$. Simplifying in stages, this gives us $\gamma(\mathrm{R}_f - 1)BV\lambda^{\frac{1}{\mathrm{a}}} - \gamma(\mathrm{R}_f - 1)RE \geq \frac{BV(\gamma(\mathrm{R}_f-1)+\delta)\lambda^{\frac{\mathrm{a}+1}{\mathrm{a}}}}{\mathrm{a}+1}$, or $\gamma(\mathrm{R}_f - 1)\lambda^{\frac{1}{\mathrm{a}}} - \gamma(\mathrm{R}_f - 1)\frac{RE}{BV} \geq \frac{(\gamma(\mathrm{R}_f-1)+\delta)\lambda^{\frac{\mathrm{a}+1}{\mathrm{a}}}}{\mathrm{a}+1}$, which is equivalent to $\gamma(\mathrm{R}_f - 1)\lambda^{\frac{1}{\mathrm{a}}} - \frac{(\gamma(\mathrm{R}_f-1)+\delta)\lambda^{\frac{\mathrm{a}+1}{\mathrm{a}}}}{\mathrm{a}+1} \geq \gamma(\mathrm{R}_f - 1)\frac{RE}{BV}$ or $(\mathrm{R}_f - 1)\lambda^{\frac{1}{\mathrm{a}}} - \frac{(\gamma(\mathrm{R}_f-1)+\delta)\lambda^{\frac{\mathrm{a}+1}{\mathrm{a}}}}{\gamma(\mathrm{a}+1)} \geq (\mathrm{R}_f - 1)\frac{RE}{BV}$. This simplifies to $(\mathrm{R}_f - 1)\lambda^{\frac{1}{\mathrm{a}}} - \frac{(\mathrm{R}_f-1)\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \geq (\mathrm{R}_f - 1)\frac{RE}{BV}$ using the definition of λ, and is equivalent to $(\mathrm{R}_f - 1)\lambda^{\frac{1}{\mathrm{a}}}\left(1 - \frac{1}{\mathrm{a}+1}\right) \geq (\mathrm{R}_f - 1)\frac{RE}{BV}$, or $\frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \geq \frac{RE}{BV}$. Thus, $TR(d_{\mathrm{L}}) \geq$

$TR(0) \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \geq \frac{RE}{BV}$, and hence $TR(0) < TR(d_{\mathrm{L}}) \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} < \frac{RE}{BV}$. Additionally, repeating the steps showing that $TR(d_{\mathrm{L}}) \geq TR(0) \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \geq \frac{RE}{BV}$ while replacing each weak inequality with a strict inequality yields no contradictions, and hence $TR(d_{\mathrm{L}}) > TR(0) \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} > \frac{RE}{BV}$ also holds, and thus so does $TR(d_{\mathrm{L}}) \leq TR(0) \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \leq \frac{RE}{BV}$. Then since $TR(d_{\mathrm{L}}) \leq TR(0) \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \leq \frac{RE}{BV}$ and $TR(d_{\mathrm{L}}) \geq TR(0) \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} \geq \frac{RE}{BV}$, we have that $TR(d_{\mathrm{L}}) = TR(0) \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} = \frac{RE}{BV}$. Notice that since D^* is non-empty and $\mathrm{D}^* \subseteq \{0, d_{\mathrm{L}}\}$ and $d^* \in \mathrm{D}^* \Rightarrow \forall d \in [0, BV], TR(d^*) \geq TR(d)$, it follows that $TR(d_{\mathrm{L}}) > TR(0) \Leftrightarrow \mathrm{D}^* = \{d_{\mathrm{L}}\}$, and $TR(d_{\mathrm{L}}) < TR(0) \Leftrightarrow \mathrm{D}^* = \{0\}$, and $TR(d_{\mathrm{L}}) = TR(0) \Leftrightarrow \mathrm{D}^* = \{0, d_{\mathrm{L}}\}$. Thus we have that $\mathrm{D}^* = \{d_{\mathrm{L}}\} \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} > \frac{RE}{BV}$, $\mathrm{D}^* = \{0\} \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} < \frac{RE}{BV}$, and $\mathrm{D}^* = \{0, d_{\mathrm{L}}\} \Leftrightarrow \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} = \frac{RE}{BV}$. This then gives us that $d_{\mathrm{L}} \in \mathrm{D}^*$ if and only if we have that $\frac{RE}{BV} \leq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$, and $0 \in \mathrm{D}^*$ if and only if $\frac{RE}{BV} \geq \frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1}$, with $\mathrm{D}^* = \{0, d_{\mathrm{L}}\}$ if and only if $\frac{\mathrm{a}\lambda^{\frac{1}{\mathrm{a}}}}{\mathrm{a}+1} = \frac{RE}{BV}$, as desired.

Part C. Regime Returns and the Returns Kink

Proof of Proposition 4 (Regime Returns)

Proof: Suppose that $D_A^* \nsubseteq D_B^*$ and $D_B^* \nsubseteq D_A^*$. Then since $D_A^* \subseteq \{0, d_L\}$ and $D_B^* \subseteq \{0, d_L\}$, it follows that either $D_A^* = \{0\}$ and $D_B^* = \{d_L\}$ or $D_B^* = \{0\}$ and $D_A^* = \{d_L\}$. We show that it must be that $D_A^* = \{0\}$ and $D_B^* = \{d_L\}$. To see this, suppose by way of contradiction that $D_B^* = \{0\}$ and $D_A^* = \{d_L\}$. Then $0 \in D_B^*$ and $d_L \in D_A^*$. From Proposition 3, we know that $0 \in D_B^* \Leftrightarrow \frac{RE_B}{BV} \geq \frac{a\lambda^{\frac{1}{a}}}{a+1}$ and $d_L \in D_A^* \Leftrightarrow \frac{RE_A}{BV} \leq \frac{a\lambda^{\frac{1}{a}}}{a+1}$. Then we have that $\frac{RE_B}{BV} \geq \frac{a\lambda^{\frac{1}{a}}}{a+1} \geq \frac{RE_A}{BV}$, and so $\frac{RE_B}{BV} \geq \frac{RE_A}{BV}$, and so

$RE_B \geq RE_A$ which contradicts the assumption that $RE_A > RE_B$. Then it can't be that $D_B^* = \{0\}$ and $D_A^* = \{d_L\}$, and so we have that $D_A^* = \{0\}$ and $D_B^* = \{d_L\}$, as desired.

We next calculate TR_A^* and AR_A^*, along with TR_B^* and AR_B^*. We start with TR_A^* and AR_A^*. Note that since $D_A^* = \{0\}$, we have that $TR_A^* = TR_A(0) = \int_0^{BV} f(x)dx = \int_0^{BV} (R_f - 1)(1-\gamma) + (\gamma(R_f - 1) + \delta)\left(\frac{x}{BV}\right)^a dx = (R_f - 1)(1-\gamma)x + \frac{BV(\gamma(R_f-1)+\delta)\left(\frac{x}{BV}\right)^{a+1}}{a+1}\Bigg|_0^{BV} = (R_f - 1)(1 - \gamma)BV + \frac{BV(\gamma(R_f-1)+\delta)\left(\frac{BV}{BV}\right)^{a+1}}{a+1} - (R_f - 1)(1-\gamma)(0) - \frac{BV(\gamma(R_f-1)+\delta)\left(\frac{0}{BV}\right)^{a+1}}{a+1} = (R_f - 1)(1 - \gamma)BV + \frac{BV(\gamma(R_f-1)+\delta)}{a+1} = BV\left((R_f - 1)(1-\gamma) + \frac{(\gamma(R_f-1)+\delta)}{a+1}\right)$. Then we have that $TR_A^* = BV\left((R_f - 1)(1-\gamma) + \frac{(\gamma(R_f-1)+\delta)}{a+1}\right)$, and since the entire return is generated internally within the firm, we have that $AR_A^* = (R_f - 1)(1-\gamma) + \frac{(\gamma(R_f-1)+\delta)}{a+1}$.

Next, we consider TR_B^* and AR_B^*. Recall from the proof of Proposition 3 that $d_L \leq RE_B \Rightarrow \frac{RE_B}{BV} > \frac{a\lambda^{\frac{1}{a}}}{a+1}$, and so the contrapositive $\frac{RE_B}{BV} \leq \frac{a\lambda^{\frac{1}{a}}}{a+1} \Rightarrow d_L > RE_B$ is true as well. Note that since $D_B^* = \{d_L\}$, we have that $d_L \in D^*$, and thus by Proposition 3 we have $\frac{RE}{BV} \leq \frac{a\lambda^{\frac{1}{a}}}{a+1}$, and hence we have that $d_L > RE_B$. Additionally, by Proposition 3, we have that $d_L = BV\lambda^{\frac{1}{a}}$. Then $TR_B^* = TR_B(d_L) = \int_0^{d_L} g(x)dx + \int_{d_L}^{BV} f(x)dx$, where

$$\int_0^{d_L} g(x)dx = \int_0^{RE_B} g(x)dx + \int_{RE_B}^{d_L} g(x)dx = \int_0^{RE_B} (R_f - 1)(1-\gamma)dx + \int_{RE_B}^{d_L} R_f - 1dx = (R_f - 1)(1-\gamma)x\Bigg|_0^{RE_B} + (R_f - 1)x\Bigg|_{RE_B}^{d_L} = (R_f - 1)(1-\gamma)RE_B - (R_f - 1)(1-\gamma)(0) + (R_f - 1)d_L - (R_f - 1)RE_B = (R_f - 1)d_L - \gamma(R_f - 1)RE_B =$$

$\left(R_f-1\right)BV\lambda^{\frac{1}{a}}-\gamma\left(R_f-1\right)RE_B$, and $\int_{d_L}^{BV} f(x)dx=\int_{d_L}^{BV}\left(R_f-1\right)(1-\gamma)+\left(\gamma\left(R_f-1\right)+\delta\right)\left(\frac{x}{BV}\right)^a dx=\left(R_f-1\right)(1-\gamma)x+\frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)\left(\frac{x}{BV}\right)^{a+1}}{a+1}\Bigg|_{d_L}^{BV}=\left(R_f-1\right)(1-\gamma)BV+\frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)\left(\frac{BV}{BV}\right)^{a+1}}{a+1}-\left(R_f-1\right)(1-\gamma)d_L-\frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)\left(\frac{d_L}{BV}\right)^{a+1}}{a+1}=\left(R_f-1\right)(1-\gamma)\left(BV-BV\lambda^{\frac{1}{a}}\right)+\frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\left(\frac{BV\lambda^{\frac{1}{a}}}{BV}\right)^{a+1}\right)=BV\left(R_f-1\right)(1-\gamma)\left(1-\lambda^{\frac{1}{a}}\right)+\frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\lambda^{\frac{a+1}{a}}\right)=BV\left(\left(R_f-1\right)(1-\gamma)\left(1-\lambda^{\frac{1}{a}}\right)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\lambda^{\frac{a+1}{a}}\right)\right)$. Then $\int_0^{d_L} g(x)dx+\int_{d_L}^{BV} f(x)dx=\left(R_f-1\right)BV\lambda^{\frac{1}{a}}-\gamma\left(R_f-1\right)RE_B+BV\left(\left(R_f-1\right)(1-\gamma)\left(1-\lambda^{\frac{1}{a}}\right)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\lambda^{\frac{a+1}{a}}\right)\right)=BV\left(\left(R_f-1\right)\lambda^{\frac{1}{a}}-\gamma\left(R_f-1\right)\frac{RE_B}{BV}+\left(R_f-1\right)(1-\gamma)\left(1-\lambda^{\frac{1}{a}}\right)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\lambda^{\frac{a+1}{a}}\right)\right)$, and so $TR_B^*=BV\left(\left(R_f-1\right)\lambda^{\frac{1}{a}}-\gamma\left(R_f-1\right)\frac{RE_B}{BV}+\left(R_f-1\right)(1-\gamma)\left(1-\lambda^{\frac{1}{a}}\right)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\lambda^{\frac{a+1}{a}}\right)\right)$. Additionally, we have that $AR_B^*=\frac{\int_{d_L}^{BV} f(x)dx}{BV-d_L}=\frac{BV\left(\left(R_f-1\right)(1-\gamma)\left(1-\lambda^{\frac{1}{a}}\right)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\lambda^{\frac{a+1}{a}}\right)\right)}{BV-BV\ ^{\frac{1}{a}}}=\frac{BV\left(\left(R_f-1\right)(1-\gamma)\left(1-\lambda^{\frac{1}{a}}\right)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\lambda^{\frac{a+1}{a}}\right)\right)}{BV\left(1-\lambda^{\frac{1}{a}}\right)}=\frac{\left(R_f-1\right)(1-\gamma)\left(1-\lambda^{\frac{1}{a}}\right)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1-\lambda^{\frac{a+1}{a}}\right)}{1-\lambda^{\frac{1}{a}}}=\left(R_f-1\right)(1-\gamma)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)\left(1-\lambda^{\frac{a+1}{a}}\right)}{(a+1)\left(1-\lambda^{\frac{1}{a}}\right)}$, so $AR_B^*=\left(R_f-1\right)(1-\gamma)+\frac{\left(\gamma\left(R_f-1\right)+\delta\right)\left(1-\lambda^{\frac{a+1}{a}}\right)}{(a+1)\left(1-\lambda^{\frac{1}{a}}\right)}$.

It then remains to compare the calculated values. For total returns, $TR_B^* > TR_A^*$ if and only if $BV\left(\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_B}{BV} + \left(R_f - 1\right)(1-\gamma)\left(1 - \lambda^{\frac{1}{a}}\right) + \frac{\left(\gamma\left(R_f - 1\right)+\delta\right)}{a+1}\left(1 - \lambda^{\frac{a+1}{a}}\right)\right) > BV\left(\left(R_f - 1\right)(1-\gamma) + \frac{\left(\gamma\left(R_f - 1\right)+\delta\right)}{a+1}\right)$, which is equivalent to $\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_B}{BV} + \left(R_f - 1\right)(1-\gamma)\left(1 - \lambda^{\frac{1}{a}}\right) + \frac{\left(\gamma\left(R_f - 1\right)+\delta\right)}{a+1}\left(1 - \lambda^{\frac{a+1}{a}}\right) > \left(R_f - 1\right)(1-\gamma) + \frac{\left(\gamma\left(R_f - 1\right)+\delta\right)}{a+1}$, which, rearranging, gives us $\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_B}{BV} + \left(R_f - 1\right)(1-\gamma)\left(-\lambda^{\frac{1}{a}}\right) + \frac{\left(\gamma\left(R_f - 1\right)+\delta\right)}{a+1}\left(-\lambda^{\frac{a+1}{a}}\right) > 0$, or $\left(R_f - 1\right)\lambda^{\frac{1}{a}} > \gamma\left(R_f - 1\right)\frac{RE_B}{BV} + \left(R_f - 1\right)(1-\gamma)\lambda^{\frac{1}{a}} + \frac{\left(\gamma\left(R_f - 1\right)+\delta\right)\lambda^{\frac{a+1}{a}}}{a+1}$, which simplifies to $\left(R_f - 1\right)\lambda^{\frac{1}{a}} > \gamma\left(R_f - 1\right)\frac{RE_B}{BV} + \left(R_f - 1\right)(1-\gamma)\lambda^{\frac{1}{a}} + \frac{\gamma\left(R_f - 1\right)\lambda^{\frac{1}{a}}}{a+1}$ by applying the definition of λ. This can then be arranged into $\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \left(R_f - 1\right)(1-\gamma)\lambda^{\frac{1}{a}} > \gamma\left(R_f - 1\right)\frac{RE_B}{BV} + \frac{\gamma\left(R_f - 1\right)\lambda^{\frac{1}{a}}}{a+1}$, which is equivalent to $\gamma\left(R_f - 1\right)\lambda^{\frac{1}{a}} > \gamma\left(R_f - 1\right)\frac{RE_B}{BV} + \frac{\gamma\left(R_f - 1\right)\lambda^{\frac{1}{a}}}{a+1}$. But recall that $\frac{RE_B}{BV} \le \frac{a\lambda^{\frac{1}{a}}}{a+1}$ and note that since $a > 0$, $\frac{a}{a+1} < 1$, and so $\frac{a\lambda^{\frac{1}{a}}}{a+1} < \lambda^{\frac{1}{a}}$, and so $\frac{RE_B}{BV} \le \frac{a\lambda^{\frac{1}{a}}}{a+1} < \lambda^{\frac{1}{a}}$, and thus $\frac{RE_B}{BV} < \lambda^{\frac{1}{a}}$. Then we have that

$\frac{RE_B}{BV} < \lambda^{\frac{1}{a}} \Rightarrow \gamma\left(R_f - 1\right)\frac{RE_B}{BV} < \gamma\left(R_f - 1\right)\lambda^{\frac{1}{a}}$, and hence $TR_B^* > TR_A^*$ if and only if

$\gamma\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_B}{BV} > \frac{\gamma\left(R_f - 1\right)\lambda^{\frac{1}{a}}}{a+1}$, where the left side is positive. This simplifies to $\lambda^{\frac{1}{a}} - \frac{RE_B}{BV} > \frac{\lambda^{\frac{1}{a}}}{a+1}$, or $\lambda^{\frac{1}{a}}\left(1 - \frac{1}{a+1}\right) > \frac{RE_B}{BV}$, or $\lambda^{\frac{1}{a}}\left(\frac{a}{a+1}\right) > \frac{RE_B}{BV}$, which rearranges to $\frac{RE_B}{BV} < \frac{a\lambda^{\frac{1}{a}}}{a+1}$. Recall from the proof of Proposition 3 that it was shown that $D_B^* = \{d_L\} \Leftrightarrow \frac{a\lambda^{\frac{1}{a}}}{a+1} >$

$\frac{RE_B}{BV}$. Then since it was already shown that $D_B^* = \{d_L\}$, we have that $\frac{a\lambda^{\frac{1}{a}}}{a+1} > \frac{RE_B}{BV}$, and hence $TR_B^* > TR_A^*$, as desired.

As for the average internal rates of return, $AR_B^* > AR_A^*$ if and only if , $AR_B^* = (R_f - 1)(1-\gamma) + \frac{(\gamma(R_f-1)+\delta)\left(1-\lambda^{\frac{a+1}{a}}\right)}{(a+1)\left(1-\lambda^{\frac{1}{a}}\right)} > (R_f - 1)(1-\gamma) + \frac{(\gamma(R_f-1)+\delta)}{a+1} = AR_A^*$. This is equivalent to $\frac{(\gamma(R_f-1)+\delta)\left(1-\lambda^{\frac{a+1}{a}}\right)}{(a+1)\left(1-\lambda^{\frac{1}{a}}\right)} > \frac{(\gamma(R_f-1)+\delta)}{a+1}$, or $\frac{\left(1-\lambda^{\frac{a+1}{a}}\right)}{\left(1-\lambda^{\frac{1}{a}}\right)} > 1$, or $1 - \lambda^{\frac{a+1}{a}} > 1 - \lambda^{\frac{1}{a}}$, or $\lambda^{\frac{a+1}{a}} < \lambda^{\frac{1}{a}}$, which rearranges to $\lambda\left(\lambda^{\frac{1}{a}}\right) < \lambda^{\frac{1}{a}}$, or $\lambda < 1$. Since $\lambda = \frac{(R_f-1)\gamma}{(R_f-1)\gamma+\delta}$ with $\delta > 0$, we have that $\lambda < 1$ and so $AR_B^* > AR_A^*$, as desired.

Proof of Proposition 5 (Kinked Returns)

Proof: We begin by showing that $TR^*(RE)$ is continuous over its domain $[0, BV]$. Note that since d is chosen over constant domain of $[0, BV]$ regardless of the value of RE (holding BV fixed), and since we have $BV > 0$ such that $[0, BV]$ is compact and non-empty $\forall\ RE \in [0, BV]$, it follows from Berge's Theorem of the Maximum that it suffices to show that the function $TR(d, RE)$ is jointly continuous in d and RE. Recall from the proof of Proposition 2 that $TR(d)$ is continuous in d over the domain $[0, BV]$. Further recall that $TR(d) = \int_d^{BV} f(x)dx + \int_0^d g(x)dx = F(d) + G(d)$, where $F(d)$ and $G(d)$ are each continuous in d over the domain $[0, BV]$ and that $f(x) = (R_f - 1)(1-\gamma) - \sigma + (\gamma(R_f - 1) + \delta + \sigma)\left(\frac{x}{BV}\right)^a$. Note, that holding BV fixed, RE does not appear in $F(d)$. Since $F(d)$ is continuous in d and unaffected by RE, it follows that $F(d, RE)$ is jointly continuous in d and RE over the interval $[0, BV]$. Then, since $TR(d, RE) = F(d, RE) + G(d, RE)$, we have that in order prove the joint continuity of

$TR(d, RE)$ in d and RE, it suffices to show that $TR(d, RE)$ is the sum of jointly continuous functions by showing that $G(d, RE)$ is jointly continuous in d and RE. Note that $G(d, RE) = \int_0^d g(x)dx$, where $g(x)$ is equal to $(R_f - 1)(1-\gamma)$ for $x \leq RE$ and equal to $(R_f - 1)$ for $x > RE$. Then, $G(d, RE) = \int_0^d (R_f - 1)(1-\gamma)dx = d(R_f - 1)(1-\gamma)\ \forall d \leq RE$, and $G(d, RE) = \int_0^{RE} (R_f - 1)(1-\gamma)dx + \int_{RE}^d (R_f - 1)dx = RE(R_f - 1)(1-\gamma) + (d - RE)(R_f - 1)\ \forall d > RE$. Then observe that this simplifies to $G(d, RE) = min\left(d(R_f - 1)(1-\gamma), RE(R_f - 1)(1-\gamma)\right) + max\left(0, (d - RE)(R_f - 1)\right)$. Since each individual argument function is jointly continuous in d and RE, and the minimum and maximum of jointly continuous functions is jointly continuous, it follows that $G(d, RE)$ is the sum of jointly continuous functions, and so is jointly continuous in d and RE. It then follows that $TR(d, RE)$ is the sum of jointly continuous functions and so is jointly continuous in d and RE. It then follows by Berge's Theorem of the Maximum that $TR^*(RE)$ is continuous over its domain $[0, BV]$, as desired.

We now prove that $TR^*(RE)$ is kinked at the point $RE = BV\frac{a\lambda^{\frac{1}{a}}}{a+1}$, such that $\lim_{h\to 0^-}\frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} \neq \lim_{h\to 0^+}\frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h}$, with $K_{TR} = \lim_{h\to 0^+}\frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} - \lim_{h\to 0^-}\frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = \gamma(R_f - 1)$. First, let $m > 0$. Now consider three otherwise-identical firms, Firm A, Firm B, and Firm I, differing only in their retained earnings relative to contributed capital. In particular, let $BV_A = BV_B = BV_I = BV$, with $RE_A = RE_I + m$, and $RE_B = RE_I - m$, with $0 < m < RE_B < RE_I < RE_A < BV$, where Firm I represents a firm where the investor is indifferent between retention and the Access Distribution, with $\frac{RE_I}{BV} = \frac{a\lambda^{\frac{1}{a}}}{a+1}$. Then by Proposition 3, $D_I^* = \{0, d_L\}$. Additionally, we have that $\frac{RE_A}{BV} > \frac{a\lambda^{\frac{1}{a}}}{a+1}$, and so by Proposition 3 $D_A^* = \{0\}$, and we have that $\frac{RE_B}{BV} < \frac{a\lambda^{\frac{1}{a}}}{a+1}$, and so by Proposition 3 $D_B^* = \{d_L\}$. Then, by Proposition 4, we have that $TR_B^* > TR_A^*$, where $TR_A^* = BV\left(\left(R_f - 1\right)(1-\gamma) + \frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\right)$ and $TR_B^* = BV\left(\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_B}{BV} + \left(R_f - 1\right)(1-\gamma)\left(1 - \lambda^{\frac{1}{a}}\right) + \frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\left(1 - \lambda^{\frac{a+1}{a}}\right)\right)$. We now calculate TR_I^*. Note that by Proposition 3, since $D_I^* = \{0, d_L\}$, we have that $TR_I(0) = TR_I(d_L) = TR_I^*$. Then note that $TR_I(0) = \int_0^{BV} f(x)dx = \int_0^{BV}\left(R_f - 1\right)(1-\gamma) + \left(\gamma\left(R_f - 1\right) + \delta\right)\left(\frac{x}{BV}\right)^a dx = \left(R_f - 1\right)(1-\gamma)x + \frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)\left(\frac{x}{BV}\right)^{a+1}}{a+1}\Bigg|_0^{BV} = \left(R_f - 1\right)(1-\gamma)BV + \frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)\left(\frac{BV}{BV}\right)^{a+1}}{a+1} - \left(R_f - 1\right)(1-\gamma)(0) - \frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)\left(\frac{0}{BV}\right)^{a+1}}{a+1} = \left(R_f - 1\right)(1-\gamma)BV + \frac{BV\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1} = BV\left(\left(R_f - 1\right)(1-\gamma) + \frac{\left(\gamma\left(R_f-1\right)+\delta\right)}{a+1}\right) = TR_A^*$. Then we have that $TR_I^* = TR_A^*$. Now let $h = m$, and consider the limit $\lim_{h\to 0^+} \frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right) - TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h}$. Observe that this can be rewritten as $\lim_{h\to 0^+} \frac{TR^*(RE_A) - TR^*(RE_I)}{h} = \lim_{h\to 0^+} \frac{TR_A^* - TR_I^*}{h} = \lim_{h\to 0^+} \frac{0}{h}$. Then let $\varepsilon > 0$. Then choose any value $z > 0$. Then note that if $0 < h < h + z$, then $\left|\frac{0}{h} - 0\right| = 0 < \varepsilon$. Thus, definitionally, $\lim_{h\to 0^+} \frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right) - T\ ^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = \lim_{h\to 0^+} \frac{0}{h} = 0$. This establishes the value of the righthand limit.

Now note that we can rewrite TR_B^* using the fact that $RE_B = RE_I - m$. This yields $TR_B^* = BV\left(\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_I - m}{BV} + \left(R_f - 1\right)(1-\gamma)\left(1 - \lambda^{\frac{1}{a}}\right) + \frac{\left(\gamma\left(R_f - 1\right) + \delta\right)}{a+1}\left(1 - \lambda^{\frac{a+1}{a}}\right)\right)$. Recall that by Proposition 3, we have that $TR_I(d_L) = TR_I^*$. Then, note that the calculation of this value follows the calculation for TR_B^* found in the proof of Proposition 4, replacing all instances of RE_B with RE_I. Thus we have that $TR_I^* = BV\left(\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_I}{BV} + \left(R_f - 1\right)(1-\gamma)\left(1 - \lambda^{\frac{1}{a}}\right) + \frac{\left(\gamma\left(R_f - 1\right) + \delta\right)}{a+1}\left(1 - \lambda^{\frac{a+1}{a}}\right)\right)$. It remains to compare the computed value for TR_I^* to the computed value for TR_B^* by calculating $TR_B^* - TR_I^*$. This yields: $TR_B^* - TR_I^* = BV\left(\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_I - m}{BV} + \left(R_f - 1\right)(1-\gamma)\left(1 - \lambda^{\frac{1}{a}}\right) + \frac{\left(\gamma\left(R_f - 1\right) + \delta\right)}{a+1}\left(1 - \lambda^{\frac{a+1}{a}}\right)\right) - BV\left(\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_I}{BV} + \left(R_f - 1\right)(1-\gamma)\left(1 - \lambda^{\frac{1}{a}}\right) + \frac{\left(\gamma\left(R_f - 1\right) + \delta\right)}{a+1}\left(1 - \lambda^{\frac{a+1}{a}}\right)\right) = BV\left(\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_I - m}{BV}\right) - BV\left(\left(R_f - 1\right)\lambda^{\frac{1}{a}} - \gamma\left(R_f - 1\right)\frac{RE_I}{BV}\right) = BV\left(-\gamma\left(R_f - 1\right)\frac{RE_I - m}{BV}\right) - BV\left(-\gamma\left(R_f - 1\right)\frac{RE_I}{BV}\right) = BV\gamma\left(R_f - 1\right)\left(\frac{RE_I}{BV}\right) - BV\gamma\left(R_f - 1\right)\left(\frac{RE_I - m}{BV}\right) = BV\gamma\left(R_f - 1\right)\left(\frac{RE_I}{BV} - \frac{RE_I - m}{BV}\right) = BV\gamma\left(R_f - 1\right)\left(\frac{m}{BV}\right) = \gamma\left(R_f - 1\right)m.$

Now let $h = -m$, and consider the limit $\lim_{h\to 0^-} \frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1} + h\right) - TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h}$. Observe that this can be rewritten as $\lim_{h\to 0^-} \frac{TR^*(RE_B) - TR^*(RE_I)}{h} = \lim_{h\to 0^-} \frac{TR_B^* - TR_I^*}{h} = \lim_{h\to 0^-} \frac{\gamma\left(R_f - 1\right)m}{h} = \lim_{h\to 0^-} \frac{-\gamma\left(R_f - 1\right)h}{h}$. Then let $\varepsilon > 0$. Then choose any value $z > 0$. Now let h be such that $0 - z < h < 0$. Then $-z < h < 0$. Then consider the value $\left|\frac{-\gamma\left(R_f - 1\right)h}{h} - \left(-\gamma\left(R_f - 1\right)\right)\right| = \left|\gamma\left(R_f - 1\right) - \frac{\gamma\left(R_f - 1\right)h}{h}\right| =$

$\left|\gamma(R_f - 1) - \gamma(R_f - 1)\right| = 0 < z$. Then, definitionally,

$\lim_{h\to 0^-} \frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = \lim_{h\to 0^-} \frac{-\gamma(R_f-1)h}{h} = -\gamma(R_f - 1)$. This establishes the value of the lefthand limit. Then $\lim_{h\to 0^-} \frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = -\gamma(R_f - 1) \neq 0 =$ $\lim_{h\to 0^+} \frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h}$, and so $TR^*(RE)$ is kinked at the point $RE = BV\frac{a\lambda^{\frac{1}{a}}}{a+1}$, as desired.

Furthermore, we have that $K_{TR} = \lim_{h\to 0^+} \frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} -$ $\lim_{h\to 0^-} \frac{TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = 0 - \left(-\gamma(R_f - 1)\right) = \gamma(R_f - 1)$, as desired.

This result directly highlights the final consequences, in terms of realized returns, of the tax wedge between contributed capital and retained earnings. Even holding the firm's investment opportunities and total book value constant, optimized total returns to the investor will differ purely as a consequence of equity composition. This arises because when retained earnings are low enough to optimally induce large-scale distributions, each new dollar of retained earnings make that tax consequences of that distribution more severe. Hence, as the firm approaches the access threshold from the left, the marginal effect of another dollar of retained earnings instead of contributed capital on total returns is $-\gamma(\mathrm{R}_f - 1)$, which is the amount by which dividend taxes reduce returns on an affected dollar of equity. From the right, however, the marginal effect is zero. Once the firm's retained earnings are sufficient to make the access distribution sub-optimal, meaning that the tax costs exceed the benefits of freeing up locked-up contributed capital, additional changes to the equity composition have no impact on returns. This contrast highlights that equity composition affects returns only when it changes the optimal behavior of the firm.

Part D. From Returns to Value

The final step of this analysis is to translate the current optimized total returns to the investor into a valuation function that captures the effects of dividend taxation in the presence of a distribution ordering rule and endogenized capital allocation. To do this, we begin by adapting the valuation results of Section II and Section III into the context of the total returns model. Recall from equation (9) in Section III that the firm's price is defined by $P_t = C_t + (1-\gamma)RE_t + \alpha_1 NI_t^a + \alpha_2 \nu_t = P_t = BV_t - \gamma RE_t + \alpha_1 NI_t^a + \alpha_2 \nu_t$, where $\alpha_1 = \omega/(R_f - \omega) \geq 0$, $\alpha_2 = \frac{R_f}{(R_f - \omega)(R_f - \lambda)} > 0$, and $NI_t^a = (1-\gamma)NI_t - \left(R_f - 1\right)[C_{t-1} + (1-\gamma)RE_{t-1}]$. Here, recall that ω is a measure of the persistence of abnormal earnings with $0 \leq \omega \leq 1$, and ν_t represents other value-relevant information impacting future earnings and returns that is not captured in current earnings, returns, book value, or equity composition.

To apply these concepts to the total returns model requires merging and synthesizing concepts in a careful way. Although equation (9) values the firm as a sum of its tax-adjusted book equity, persistence-adjusted abnormal earnings, and other information, Section III does not account for the effects of endogenous allocation in the presence of a distribution ordering rule. To include these effects in the valuation, we consider the value of distributed funds separately from the value of retained funds.[28] Thus, consider a function of firm value $FV(RE) = \Psi(RE) + \Phi(RE)$, where $\Psi(RE)$ is the value of funds optimally distributed given the firm's equity

[28] In the definition of α2, λ carries its Section III meaning. This valuation can be thought of as the value of the company prior to any distributions made but conditional on optimal distribution policy. It is comparable to the price of a stock after dividends are announced but before the ex-dividend date.

composition (holding all other parameters fixed, including total BV), and $\Phi(RE)$ is the value of all funds optimally retained. Then note that since the future stream of returns for distributed funds is known and fixed, it can be valued as a simple perpetuity in the following way:

$$\Psi(RE) = \sum_{\tau=1}^{\infty} \frac{G(d^*(RE), RE)}{{R_f}^{\tau}} = \frac{G(d^*(RE), RE)}{R_f - 1}$$

Here, $G(d^*(RE), RE)$ is the total return to the investor each period on the original principal of the optimally distributed funds in the initial period, where optimal distribution decisions vary as function of equity composition, holding other parameters fixed. In order for $d^*(RE)$ to be a well-defined function given that $D^* = \{0, d_L\}$ when $RE = BV\frac{a\lambda^{\frac{1}{a}}}{a+1}$, let $d^*\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right) = 0$. $R_f - 1$ is the discount rate throughout, as Section III establishes. The value of retained funds is more complex, and relies directly on insights from Section II and Section III. In particular, the value of tax-adjusted book equity becomes the tax-adjusted value of undistributed funds, whereas the abnormal earnings component of the valuation becomes total internal returns less foregone returns on external reinvestment, adjusted by persistence. Applying this logic yields the following formulation:

$$\Phi(RE) = BV - d^*(RE) - max\big(0, \gamma\big(RE - d^*(RE)\big)\big) + \alpha_1\big(TR^*(RE) - G(BV, RE)\big)$$

This equation treats the normal earnings component of the valuation for undistributed funds as being equal to retained book value $BV - d^*(RE)$, less a tax adjustment $max\big(0, \gamma\big(RE - d^*(RE)\big)\big)$ on remaining retained earnings, if any. The abnormal earnings component, $\alpha_1\big(TR^*(RE) - G(BV, RE)\big) = \frac{\omega\left(TR^*(RE) - G(BV,RE)\right)}{R_f - \omega} = \frac{\omega\left(TR^*(RE) - \int_{d^*(RE)}^{BV} g(x)dx - \int_0^{d^*(RE)} g(x)dx\right)}{R_f - \omega}$, is equal to optimized total returns less returns to distributed funds and foregone external reinvestment returns on undistributed funds, valued as a perpetuity with decay where the rate of decay is governed by the relative persistence or persistence of abnormal returns. Combining the formulas for $\Psi(RE)$ and $\Phi(RE)$, we have that firm value is given by the following:[29]

$$FV(RE) = \frac{G(d^*(RE), RE)}{R_f - 1} + BV - d^*(RE) - max\big(0, \gamma\big(RE - d^*(RE)\big)\big)$$
$$+ \alpha_1\big(TR^*(RE) - G(BV, RE)\big)$$

This valuation function can be further simplified. Note that $\frac{G(d^*(RE),RE)}{R_f-1} = \frac{\int_0^{d^*(RE)} g(x)dx}{R_f-1} = \frac{(R_f-1)(1-\gamma)min(RE,d^*(RE))+(R_f-1)max(0,d^*(RE)-RE)}{R_f-1} = (1-\gamma)min\big(RE, d^*(RE)\big) + max(0, d^*(RE) - RE)$. Then the firm value function be written as: $FV(RE) = (1 - \gamma)min\big(RE, d^*(RE)\big) + max(0, d^*(RE) - RE) + BV - d^*(RE) - max\big(0, \gamma\big(RE - d^*(RE)\big)\big) + \alpha_1\big(TR^*(RE) - G(BV, RE)\big)$. Note that if $d^*(RE) = RE$, this simplifies to $FV(RE) = (1 - \gamma)RE + BV - RE + \alpha_1\big(TR^*(RE) - G(BV, RE)\big) = BV - \gamma RE + \alpha_1\big(TR^*(RE) - G(BV, RE)\big)$, where $TR^*(RE) - G(BV, RE)$ is a measure of abnormal earnings. This is equivalent to the equation (9) definition of value while allowing an initial allocation decision to be formed endogenously in the presence of a distribution ordering rule. Note that for $d^*(RE) < RE$, we have $FV(RE) = (1 - \gamma)d^*(RE) + BV - d^*(RE) - \gamma\big(RE - d^*(RE)\big) + \alpha_1\big(TR^*(RE) - G(BV, RE)\big)$, which again simplifies to $FV(RE) = BV - \gamma RE + \alpha_1\big(TR^*(RE) - G(BV, RE)\big)$. Similarly, for $d^*(RE) > RE$, we have $FV(RE) = (1 - \gamma)RE + d^*(RE) - RE + BV - d^*(RE) + \alpha_1\big(TR^*(RE) - G(BV, RE)\big)$, which likewise simplifies to

[29] Three details of this valuation model warrant explication, set out under Remarks on the Valuation Function below.

$FV(RE) = BV - \gamma RE + \alpha_1\big(TR^*(RE) - G(BV, RE)\big)$. Thus firm value can cleanly be stated as the sum of tax adjusted book value and persistence-adjusted abnormal earnings. It remains to show how previous results regarding returns translate into value in that setting, and how the new valuation insights differ from the linear model. The main insights are collected in Proposition 6, whose proof follows.

Remarks on the Valuation Function

There are three details of this valuation model worth additional explication. First, the abnormal earnings component excludes any equivalent of the term $\alpha_2 v_t$ from equation (9). This is a mathematical simplification. Because this term is by construction independent of initial total returns, equity composition, and other aspects of this analysis, it has no qualitatively important role in the analysis. The valuation model can thus be thought of as representing the case where all value-relevant information is contained in current returns and equity composition. Second, because the valuation function follows the convention from Part B onward where $\sigma = 0$, the valuation function excludes funds distributed as part of the Baseline Distribution. Given that the funds associated with the Baseline Distribution will always be distributed regardless of equity composition and will always be valued as a simple perpetuity, requiring $\sigma = 0$ simplifies the analysis without sacrificing meaningful insight. The result is the valuation function is best interpreted as applying to all the firm's equity except funds guaranteed to be distributed under the Baseline Distribution. Finally, differing from the definition of abnormal returns given in Section II, the valuation model does not discount returns on undistributed funds by the tax rate (since within the total returns model, no tax is levied unless a distribution is made). While this distinction may seem significant, it is fundamentally notational in nature. Adjusting the internal returns function $f_{new}(0) = R_f - 1 - \sigma$ and $f_{new}(BV) = \frac{R_f - 1}{1-\gamma} + \delta$ would yield identical qualitative insights throughout Appendix B and result in a valuation function where returns on undistributed funds are tax-discounted. This is because the driving force behind the results in Appendix B is the fact that $HurdleRate_{RE} = HurdleRate_C * (1 - \gamma)$, which is robust to a wide array of tax assumptions, including whether the assumed eventual tax on internally generated funds is explicitly included or not.

Proof of Proposition 6 (Kinked Value)

Proof: We begin by showing that $FV(RE)$ is continuous in RE over $[0, BV]$. Recall from the proof of Proposition 5 that the function $G(d, RE)$ is jointly continuous. Then, holding the first argument fixed at $d = BV$, it follows that $G(BV, RE)$ is continuous over $[0, BV]$. Then, since $-\alpha_1$ is a scalar, it follows that $-\alpha_1 G(BV, RE)$ is continuous over $[0, BV]$ as well. Next, note that BV is being held fixed, it is constant in RE and therefore continuous. Then $BV - \gamma RE$ is merely the sum of a constant and a continuous function and is therefore continuous as well. Additionally, by Proposition 5, $TR^*(RE)$ is continuous in RE over $[0, BV]$, and since α_1 is a scalar, it follows that $\alpha_1 TR^*(RE)$ is continuous over $[0, BV]$ as well. Then $FV(RE)$ is a sum of continuous functions and hence is continuous over $[0, BV]$, as desired.

Next we show that $FV(RE)$ is kinked at the threshold point $RE = BV\frac{a\lambda^{\frac{1}{a}}}{a+1}$, with $K_{FV} = \lim_{h\to 0^+} \frac{FV\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right) - FV\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} - \lim_{h\to 0^-} \frac{FV\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right) - FV\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = \alpha_1\gamma\left(R_f - 1\right)$. First, consider the righthand limit, $\lim_{h\to 0^+} \frac{FV\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}+h\right) - FV\left(BV\frac{a\lambda^{\frac{1}{a}}}{a+1}\right)}{h}$. Note that since $FV(RE) = \alpha_1 TR^*(RE) - \alpha_1 G(BV, RE) + BV - \gamma RE$, this can be rewritten as

$$\lim_{h\to 0^+}\frac{\alpha_1 TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-\alpha_1 G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)+BV-\gamma\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-\alpha_1 TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)+\alpha_1 G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)-BV+\gamma\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)}{h}=$$

$$\lim_{h\to 0^+}\frac{\alpha_1 TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-\alpha_1 G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-\gamma\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-\alpha_1 TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)+\alpha_1 G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)+\gamma\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)}{h}=$$

$$\lim_{h\to 0^+}\frac{\alpha_1 TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-\alpha_1 TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)+\gamma\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}-BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}-h\right)+\alpha_1\left(G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)-G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)\right)}{h}=$$

$$\lim_{h\to 0^+}\frac{\alpha_1 TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-\alpha_1 TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)-\gamma h+\alpha_1\left(G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)-G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)\right)}{h}=$$

$$\alpha_1\lim_{h\to 0^+}\frac{TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)}{h}-\gamma\lim_{h\to 0^+}\frac{h}{h}+$$

$$\alpha_1\lim_{h\to 0^+}\frac{G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)-G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)}{h}=\alpha_1\lim_{h\to 0^+}\frac{TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)}{h}+$$

$\alpha_1\lim_{h\to 0^+}\frac{G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)-G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)}{h}-\gamma$. Applying Proposition 5, the first term is equal to zero. As for the second term, it can be calculated as $\alpha_1\lim_{h\to 0^+}\frac{G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)-G\left(BV,\ BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)}{h}=$

$$\alpha_1\lim_{h\to 0^+}\frac{(R_f-1)(1-\gamma)BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+(R_f-1)\left(BV-BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)-(R_f-1)(1-\gamma)\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-(R_f-1)\left(BV-\quad\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}-h\right)}{h}=$$

$\alpha_1\lim_{h\to 0^+}\frac{h(R_f-1)-h(R_f-1)(1-\gamma)}{h}=\alpha_1\lim_{h\to 0^+}\frac{h\gamma(R_f-1)}{h}=\ \alpha_1\gamma\left(R_f-1\right)$. Then, combining terms, we have $\lim_{h\to 0^+}\frac{FV\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}+h\right)-FV\left(BV\frac{\alpha\lambda^{\frac{1}{\alpha}}}{\alpha+1}\right)}{h}=0+\alpha_1\gamma\left(R_f-1\right)-\gamma=\alpha_1\gamma\left(R_f-1\right)-\gamma$. This establishes the righthand limit.

Now consider the lefthand limit. Repeating the above algebra, we obtain that

$$\lim_{h\to 0^-} \frac{FV\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}+h\right)-FV\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = \alpha_1 \lim_{h\to 0^-} \frac{TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}\right)}{h} + \alpha_1 \lim_{h\to 0^-} \frac{h\gamma(R_f-1)}{h} -$$

$$\gamma \lim_{h\to 0^-} \frac{h}{h} = \alpha_1 \lim_{h\to 0^-} \frac{TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}\right)}{h} - \gamma + \alpha_1\gamma(R_f - 1)$$. Then, applying Proposition 5,

we have that $\lim_{h\to 0^-} \frac{TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}+h\right)-TR^*\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = -\gamma(R_f - 1)$. Then we have that

$\lim_{h\to 0^-} \frac{FV\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}+h\right)-FV\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = -\alpha_1\gamma(R_f - 1) - \gamma + \alpha_1\gamma(R_f - 1) = -\gamma$. This establishes the lefthand limit.

We can now calculate the kink. This is given by $K_{FV} = \lim_{h\to 0^+} \frac{FV\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}+h\right)-FV\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}\right)}{h} -$

$\lim_{h\to 0^-} \frac{FV\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}+h\right)-FV\left(BV\frac{\alpha\lambda^{\frac{1}{a}}}{a+1}\right)}{h} = \alpha_1\gamma(R_f - 1) - \gamma - (-\gamma) = \alpha_1\gamma(R_f - 1) - \gamma + \gamma =$

$\alpha_1\gamma(R_f - 1)$, as desired.

This shows that the kink in returns flows through to value when using the above valuation model, with $K_{FV} = \alpha_1 K_{TR}$ In particular, it is of note that with perfect persistence in the form of $\omega = 1$, we have that $K_{FV} = \gamma$, such that the kink is a simple function of taxes.

Part E. The Kink under Threshold Uncertainty

By Propositions 5 and 6, proved in Parts C and D above, for a known threshold the value function can be written

$$V(RE;\, RE^*) \;=\; A \;+\; \gamma(RE^* \;-\; RE)^+,$$

where A is the retention-regime level and $(x)^+$ denotes $\max(x, 0)$: slope $-\gamma$ below RE^*, slope 0 above, the convex kink of Part D.

Proof of Corollary 1: For RE off the threshold, $\mathrm{d}V/dRE \;=\; -\gamma$ if $RE < RE^*$ and 0 if $RE > RE^*$. Slopes are bounded, so expectation and differentiation interchange, giving $\mathrm{dE}[V]/dRE \;=\; -\gamma\,\mathrm{P}(RE^* \;>\; RE) \;=\; -\gamma(1 \;-\; \mathrm{F}(RE))$. This is continuous and nondecreasing in RE, so E[V] is convex; its slope runs from $-\gamma$ to 0, so the total slope change is γ, as desired.

Resolution note: the retention level A is constant in RE by Proposition 4's retention expression, so holding fundamentals fixed is a property, not an added assumption. The intended reading of threshold uncertainty is heterogeneity in the schedule parameters or investor uncertainty about them, the reading Sections VI and VII use. The interchange of expectation and differentiation is justified by dominated convergence, since the slopes are bounded by γ. The corollary fixes the effective rate γ and lets only the threshold vary; if the effective rate itself varies across the support, the curvature scales with the relevant effective-rate blend rather than a single invariant γ, the qualification noted in Section VI.E.